Calculation of the Maximum Force Distribution of an Index Finger by using Liner Programming Method

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1. *Introduction*

Human finger can perfome delicate and powerful motion such as grasping and picking up some object. It is considered that kinematic redundancy and muscle redundancy of a finger and coordinated control of multiple joints and muscles realise these dexterous motions. Therefore, detailed analysis of the redundancies and coordinated control and mechanical characteristics of a finger could be applied to technological application like robot hand and medical application for rehabilitation of a finger.

There are many studies about a finger structure and relationship between each muscle force and output force at fingertip in static condition[1]–[9]. Many studies which have focused on the relationship between each muscle force and output force at fingertip in static condition have tried to obtain muscle forces achieving a given output force vector at fingertip. But, when we treat a finger as a manipulator and investigate its performance, it is more important to calculate the maximum output force distribution of a finger by solving forward kinematics and optimization problem than to calculate each muscle force as an input by solving inverse kinematics.

Though conventional studies have analysed by making one-to-one relationship between muscle force and output force vector, one muscle force vector does not one-to-one correspondence with one fingertip output force vector due to kinematic redundancy of finger. Therefore, it is necessary to bring extra variables like distal phalanx torque and constraints into analysis[1]. This makes analysis complicated and difficult. Additionally, the place where we can calculate output force at is limited to fingertip, we can not calculate output force at any other place such as center of a middle phalanx of an index finger by using conventional calculation system.

In this paper, the authors introduce a method for calculating the maximum output force distribution at any point of an index finger. This method is possible to analyse mechanical characteristics of finger in more detail. The method is based on a simple relationship between muscle force vector and output force vector in static condition and the problem to maximize the magnitude of output force by utilizing redundancy of a finger could be formulated as a linear programming problem. In this paper, buchner's index finger model[5] is used for specific derivation of the method. Similar method will be derived from many other finger models in static condition. Finally, the distributions of maximum output force of an index finger has been calculated by introduced method and the characteristics of an index finger in static condition have been also considered.

2. *Calculation method for the maximum output force distribution of an index finger*

2.1 *Buchner's index finger model*

At first, an index finger model is described. Proposed method does not depend on one specific model, therefore a simple index finger model which was proposed by buchner is used[5]. Fig. 1 shows a structure of an index finger. ED, FDP, FDS, Int, Lum are muscles and MB, L, T are tendons respectively.

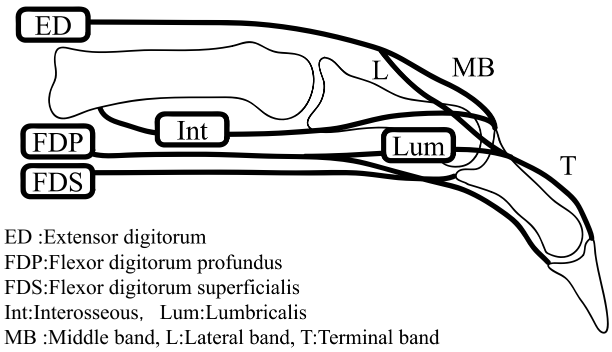

Fig. 1 Index finger model[5].

Variation length of each muscle and tendon * is defined as e_*, and let e_* be possitive when muscle and tendon * is extended and be negative when muscle and tendon * is flexed. Joint angles θ_{MP}, θ_{PIP}, θ_{DIP} are defined as shown in Fig. 2. There are following equations about joint angles and variation length of each muscle and tendon.

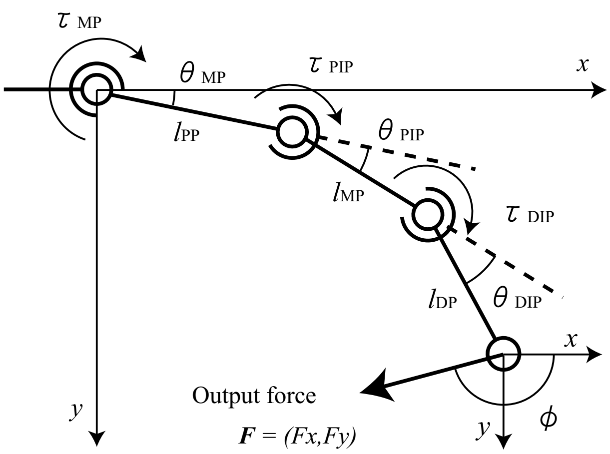

Fig. 2 Coordinate system and joint angles.

$$\begin{aligned} e_{\rm ED} &= -R_{11}\theta_{\rm MP} - R_{12}\theta_{\rm PIP} && (1)\\ e_{\rm FDP} &= (R_{21}+R'_{21}\theta_{\rm MP})\theta_{\rm MP} + (R_{22}+R'_{22}\theta_{\rm PIP})\theta_{\rm PIP} \\ &+ (R_{23}+R'_{23}\theta_{\rm DIP})\theta_{\rm DIP} && (2)\\ e_{\rm FDS} &= (R_{31}+R'_{31}\theta_{\rm MP})\theta_{\rm MP} + R_{32}\theta_{\rm PIP} && (3)\\ e_{\rm Int} &= R_{41}\theta_{\rm MP} - R_{42}\theta_{\rm PIP} && (4)\\ e_{\rm LUM} &= (R_{51}-R_{21}-R'_{21}\theta_{\rm MP})\theta_{\rm MP} \\ &- (R_{52}+R'_{52}\theta_{\rm PIP}+R_{22}+R'_{22}\theta_{\rm PIP})\theta_{\rm PIP} \\ &- (R_{13}+R_{23}+R'_{23}\theta_{\rm DIP})\theta_{\rm DIP} && (5) \end{aligned}$$

R_{ij} denotes moment arm. Table **1** shows the values of moment arm and Table **2** shows the values of link length those are published in [5].

Table 1 Moment arms[5].

i	Joint	MP		PIP		DIP	
	Muscle	[mm]		[mm]		[mm]	
		R_{i1}	R'_{i1}	R_{i2}	R'_{i2}	R_{i3}	R'_{i3}
1	ED	10	／	5	1.21	3.5	／
2	FDP	11.2	2.08	9.67	2.21	4.54	0.261
3	FDS	10	2.05	7	／	／	／
4	Int	5	／	5	／	／	／
5	Lum	11.5	／	4.34	-0.486	／	／

Table 2 Link lengths[5].

	$l_{\rm PP}$	$l_{\rm MP}$	$l_{\rm DP}$
Length [mm]	46	28	20

Equations (1)–(5) are put into a following equation:

$$\boldsymbol{E} = \boldsymbol{p}(\boldsymbol{\Theta}) \tag{6}$$

Where $\boldsymbol{E} = (e_{\rm ED}, e_{\rm FDP}, e_{\rm FDS}, e_{\rm Int}, e_{\rm Lum})^{\rm T}$, $\boldsymbol{\Theta} = (\theta_{\rm MP}, \theta_{\rm PIP}, \theta_{\rm DIP})^{\rm T}$. There is a following relationship between $\theta_{\rm DIP}$ and $\theta_{\rm PIP}$:

$$\theta_{\rm DIP} = \beta\theta_{\rm PIP}^2 \tag{7}$$

Where $\beta = R'_{12}/R_{13}$. In buchner's model, muscles and tendons are assumed not to slack. Hence, the following inequations must be satisfied:

$$0 \le f_{\rm Lum} \le f_{\rm FDP} \tag{8}$$

f_* denotes tensional force which muscle and tendon * produces.

2.2 *Relationship between muscle force vecotr $\boldsymbol{f}$ and output force vector $\boldsymbol{F}$*

In this subsection, a simple equation about muscle force $\boldsymbol{f}$ and output force $\boldsymbol{F}$ is derived[8]. The time derivative of (6) is described as follow:

$$\dot{\boldsymbol{E}} = \boldsymbol{G}(\boldsymbol{\Theta})\dot{\boldsymbol{\Theta}} \tag{9}$$

$\boldsymbol{G}(\boldsymbol{\Theta}) = [g_{ij}]$ is a 5×3 matrix and each element is as follows:

$$\begin{aligned} & g_{11} = -R_{11},\ g_{12} = -R_{12},\ g_{13} = 0,\\ & g_{21} = R_{21} + 2R'_{21}\theta_{\rm MP},\ g_{22} = R_{22} + 2R'_{22}\theta_{\rm PIP},\\ & g_{23} = R_{23} + 2R'_{23}\theta_{\rm DIP},\ g_{31} = R_{31} + 2R'_{31}\theta_{\rm MP},\\ & g_{32} = R_{32},\ g_{33} = 0,\ g_{41} = R_{41},\ g_{42} = -R_{42},\ g_{43} = 0,\\ & g_{51} = R_{51} - R_{21} - 2R'_{21}\theta_{\rm MP},\\ & g_{52} = -(R_{52} + 2R'_{52}\theta_{\rm PIP} + R_{22} + 2R'_{22}\theta_{\rm PIP}),\\ & g_{53} = -(R_{13} + R_{23} + 2R'_{23}\theta_{\rm DIP}) \end{aligned}$$

By applying the principle of virtual work to (9), the following equation is derived:

$$\boldsymbol{T} = \boldsymbol{G}(\boldsymbol{\Theta})^{\rm T}\boldsymbol{f} \tag{10}$$

Where $\boldsymbol{T} = (\tau_{\rm MP},\ \tau_{\rm PIP},\ \tau_{\rm DIP})^{\rm T}$ and τ_* is a torque of joint * and $\boldsymbol{f} = (f_{\rm ED},\ f_{\rm FDP},\ f_{\rm FDS},\ f_{\rm Int},\ f_{\rm Lum})^{\rm T}$. From (7), (9) and (10), the following equatin is obtained:

$$\boldsymbol{T}^* = \boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\rm T}\boldsymbol{f} \tag{11}$$

Where $\boldsymbol{T}^* = (\tau_{\rm MP},\ \tau_{\rm PIP} + 2\beta\theta_{\rm PIP}\tau_{\rm DIP})^{\rm T}$, $\boldsymbol{\Theta}^* = (\theta_{\rm MP}, \theta_{\rm PIP})^{\rm T}$. $\boldsymbol{G}^*(\boldsymbol{\Theta}^*) = [g^*_{ij}]$ is a 5×2 matrix and each element is as follows:

$$g^*_{i1} = g_{i1},\ g^*_{i2} = g_{i2} + 2\beta\theta_{\rm PIP}g_{i3} \quad (i = 1, \cdots, 5) \tag{12}$$

Let $\boldsymbol{x} = (x,\ y)^{\rm T}$ be a position vector that denotes any point on a finger such as a fingertip. However, it is a point to notice that $\boldsymbol{x}$ is not limited to be a fingertip. $\boldsymbol{x}$ is a function of $\boldsymbol{\Theta}$, so we have:

$$\boldsymbol{x} = \boldsymbol{L}(\boldsymbol{\Theta}) \tag{13}$$

The time derivative of (13) is:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{\Theta})\dot{\boldsymbol{\Theta}} \tag{14}$$

$\boldsymbol{J}(\boldsymbol{\Theta}) = [j_{ij}] = \partial \boldsymbol{L}/\partial \boldsymbol{\Theta}$ is a 2×3 matrix and called a jacobian matrix. By applying the principle of virtual work to (14), the following equation is derived:

$$\boldsymbol{T} = \boldsymbol{J}(\boldsymbol{\Theta})^{\mathrm{T}}\boldsymbol{F} \tag{15}$$

Where $\boldsymbol{F} = (F_x, F_y)^{\mathrm{T}}$ is a output force vector at $\boldsymbol{x}$. From (7), (14) and (15), the following equation is derived:

$$\boldsymbol{T}^* = \boldsymbol{J}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{F} \tag{16}$$

Where $\boldsymbol{J}^*(\boldsymbol{\Theta}^*) = [j^*_{ij}]$ is defined as $j^*_{i1} = j_{i1},\ j^*_{i2} = j_{i2} + 2\beta\theta_{\mathrm{PIP}} j_{i3} \quad (i = 1, 2)$. From (11) and (16), we have:

$$\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} = \boldsymbol{J}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{F} \tag{17}$$

It is obvious that muscle force vecotr $\boldsymbol{f}$ and output force vector $\boldsymbol{F}$ satisfy such a simple equation.

2.3 *Linear programming program*

When we calculate the maximum output force distribution, it is useful to transform $\boldsymbol{F}$ into the following equation with paying attention to the output force direction ϕ.

$$\boldsymbol{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = F\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \tag{18}$$

Where F is the magnitude of output force. By substituting (18) into (17), the following equation is derived:

$$\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} = F\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi) \tag{19}$$

Where $\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi) = \boldsymbol{J}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}(\cos\phi, \sin\phi)^{\mathrm{T}}$. By transforming (19) equivalently, the following two equations are derived:

$$F = \boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)^{+}\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} \tag{20}$$

$$(\boldsymbol{I} - \boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)^{+})\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} = \boldsymbol{0} \tag{21}$$

Where $\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)^{+}$ is a pseudo inverse matrix of $\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)$ and $\boldsymbol{I}$ is a 2×2 unit matrix. Equation (20) indicates that the magnitude of the output force vector F is calculated from a muscle force vector $\boldsymbol{f}$ explicitly. Equation (21) is a restraint condition of the muscle force vector $\boldsymbol{f}$ to keep finger posture. It is a big feature that both (20) and (21) are equations about linearly-transformed muscle force vector $\boldsymbol{f}$. In addition to these equations, an upper limit and a lower limit of muscle force are considered at the real model, Therefore muscle force vector $\boldsymbol{f}$ needs to satisfy the following inequality:

$$\boldsymbol{f}_{\min}(\boldsymbol{\Theta}^*) \le \boldsymbol{f} \le \boldsymbol{f}_{\max}(\boldsymbol{\Theta}^*) \tag{22}$$

Equation (22) is applied to each element respectively. We can formulate the problem to obtaion the maximum output force F to the direction of the output force ϕ as a following linear programming problem.

$$\begin{aligned} \max_{\boldsymbol{f}} \quad & F = \boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)^{+}\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} \\ \text{s.t.} \quad & (\boldsymbol{I} - \boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)^{+})\boldsymbol{G}^*(\boldsymbol{\Theta}^*)^{\mathrm{T}}\boldsymbol{f} = \boldsymbol{0} \\ & 0 \le f_{\mathrm{Lum}} \le f_{\mathrm{FDP}} \\ & \boldsymbol{f}_{\min}(\boldsymbol{\Theta}^*) \le \boldsymbol{f} \le \boldsymbol{f}_{\max}(\boldsymbol{\Theta}^*) \end{aligned}$$

2.4 *Reasons to be able to formulate the problem as a linear programming problem*

In this subsection, the reasons why we can formulate the problem to obtaion the maximum output force F to the direction of the output force ϕ as a linear programming problem is described. At first, the most important equation in this method is (17) and the key point is that muscle force vecotr $\boldsymbol{f}$ and output force vector $\boldsymbol{F}$ satisfy such a simple equation. Equations (20) and (21) which are obtained by transforming (17) equivalently enable us to formulate the problem as a linear programming problem. These equations are derived from (6) and (13), and we always have both (6) and (13) in static condition, therefore this method could be applied to many other finger model in static condition. Secondly, simplifying the problem to obtain the maximum output force $\boldsymbol{F}$ as obtaining the magnitude of the force to the direction ϕ is a point too. Specifying the direction of output force makes the problem easy, because the value to obtain becomes only the magnitude of output force. Finally, transforming (19) into (20) and (21) is a point. If the pseudo inverse matrix of $\boldsymbol{b}(\boldsymbol{\Theta}^*, \phi)$ exists, output force vector $\boldsymbol{F}$ becomes calculable by muscle force vector $\boldsymbol{f}$ by this transformation, . Due to this transformation, this method has huge advantages that where we can calculate the distribution of output force at is not limited to fingertip but to any point on a finger.

3. *Results of case studies*

In this section, the method for calculating the maximum output force distribution from a finger is validated. Specifically, the following two maximum output force distributions are compared. First one is calculated by the proposed method and second one is a measurement results of adults in previous research[9]. Additionally, each muscle force has been calculated when a finger produces maximum output force and also the relationship of the distribution of maximum output force and muscle forces has been considered.

3.1 *Conditions of case studies*

In this subsection, the parameters used for calculating the distribution of the maximum output force are described. At first, parameters of momet arm and link length shown in Table **1** and Table **2** are used respectively. It is assumed that the minimum muscle force $\boldsymbol{f}_{\min}(\boldsymbol{\Theta}^*)$ and the maximum muscle force $\boldsymbol{f}_{\max}(\boldsymbol{\Theta}^*)$ are supposed to be constant for ease in calculation, and the minimum muscle force is supposed to be zero vector. It is known that the maximum muscle force of each muscle is proportional to physiological cross-sectional area (PCSA)[6], therefore the maximum muscle force is supposed to be obtain as the product of PCSA shown in Table **3**[7] and constant of proportion $35\mathrm{N/cm^2}$[4].

The maximum output force distributions at fingertip in the following five cases have been calculated.

Table 3 PCSA[7]

Muscle Name	ED	FDP	FDS	Int	Lum
PCSA $[\mathrm{cm}^2]$	2.51	4.10	3.65	5.76	0.36

case (a) joint angles $\boldsymbol{\Theta} = (20^\circ,\ 20^\circ,\ 2.4^\circ)^{\mathrm{T}}$

case (b) joint angles $\boldsymbol{\Theta} = (20^\circ,\ 40^\circ,\ 9.7^\circ)^{\mathrm{T}}$

case (c) joint angles $\boldsymbol{\Theta} = (20^\circ,\ 60^\circ,\ 22^\circ)^{\mathrm{T}}$

case (d) joint angles $\boldsymbol{\Theta} = (40^\circ,\ 20^\circ,\ 2.4^\circ)^{\mathrm{T}}$

case (e) joint angles $\boldsymbol{\Theta} = (60^\circ,\ 20^\circ,\ 2.4^\circ)^{\mathrm{T}}$

The direction of output force ϕ has been changed to every $\pi/100[\mathrm{rad}]$ in order to calculate the distributions.

3.2 Calculation results

The calculated maximum output force distributions using the method in case (a), (b) and (c) are shown in Fig. 3-(1). The measured distributions are shown in Fig. 3-(2)[9]. The magnitudes of output force are shown in Fig. 4. The muscle forces on condition of maximum output force are shown in Fig. 5. Furthermore the results in case (a), (d) and (e) are shown in Fig. 6 – 8.

3.3 Comparison of measured and calculated distributions

Measured distributions in Fig. 3-(2) and Fig. 6-(2) had following features[9].

1. The distribution changed considerably with changes in the PIP and DIP joint angles rather than in the MP joint angle.
2. In case (a) and (d), a finger was able to produce a great force around the distal direction.
3. The distribution in case (c) was rounder than in case (a).
4. The magnitude of output force in the flexion direction was larger than in the extension direction in all cases.

The calculated distributions of the maximum output force shown in Fig. 3-(1) and Fig. 6-(1) had all features as mentioned above. These features also could be read in the magnitude of output force shown in Fig. 4 and Fig. 7. Consequently, it can be said that the method is a good calculation method with considerations of the mechanical characteristics of an index finger in static condition.

3.4 Relationship between muscle forces and output force

In this subsection, relationship between muscle forces and characteristics of an index finger in static condition has been examined. Fig. 5 and Fig. 8 show the muscle forces depending on the direction of the output force. At first, there are two sections broadly. In one section, muscle FDP, FDS, Int and Lum produce large force. In another section, muscle ED produces large force. The former section corresponds to the flexion direction of output force and the latter section corresponds to the extension direction of output force respectively. From Fig. 4 and Fig. 7, the magnitude of the output force in the former section is larger than in the latter section. The reason why big difference of the magnitude of output force in the flexion direction and in the extension direction exists could be conjectured to due to the difference of the number of muscle which produces large force.

In case (a), the magnitude of the force of muscle Int greatly changes along with the change of the direction of the output force in the flexion direction. This denotes muscle Int controls the direction of output force in the flexion direciton. This feature is also seen in case (d) and (e). Consequently, muscle Int controls the output force direction in the flexion direciton at extension posture such as in case (a), (d) and (e). Additionaly, the patterns of magnitudes of the output force do not change largely along with the change of joint angle MP from Fig. 6 and Fig. 7. On the other hand, coordinated control of all muscles controls the output force direction in case (c). However, the distribution of maximum output force depends on not only muscle forces but also finger posture. Hense, it is necessary to investigate the influence of change of posture especially in case (a), (b) and (c).

4. Conclusion

For the purpose of investigating finger characteristics in static condition, the authors have described the method to obtain the maximum force distribution of an index finger by using linear programming method in this paper. the problem has been formulated concretely using buchner's index finger model, and has calculated the maximum output force distributions at fingertip in some postures. It has been confirmed that the characteristics of calculated and measured distributions are similar. The relationship between the muscle forces and the output force distribution has been examined.

However, distribution of the maximum output force depends on both muscle forces and finger posture, it will needed to investigate the influence of change of posture for the future. In addition, the condition (7) that is a constraint of joint angles is so strong that we can transform a jacobian matrix $\boldsymbol{J}(\boldsymbol{\Theta})$ into a square matrix $\boldsymbol{J}^*(\boldsymbol{\Theta}^*)$. Using buchner's model as a finger model has suppressed the advantage of the method that the method is able to calculate the distribution when $\boldsymbol{J}(\boldsymbol{\Theta})$ is not a square matrix. As a future work, using another finger model and calculating the maximum output force distribution at point except fingertip will be considered.

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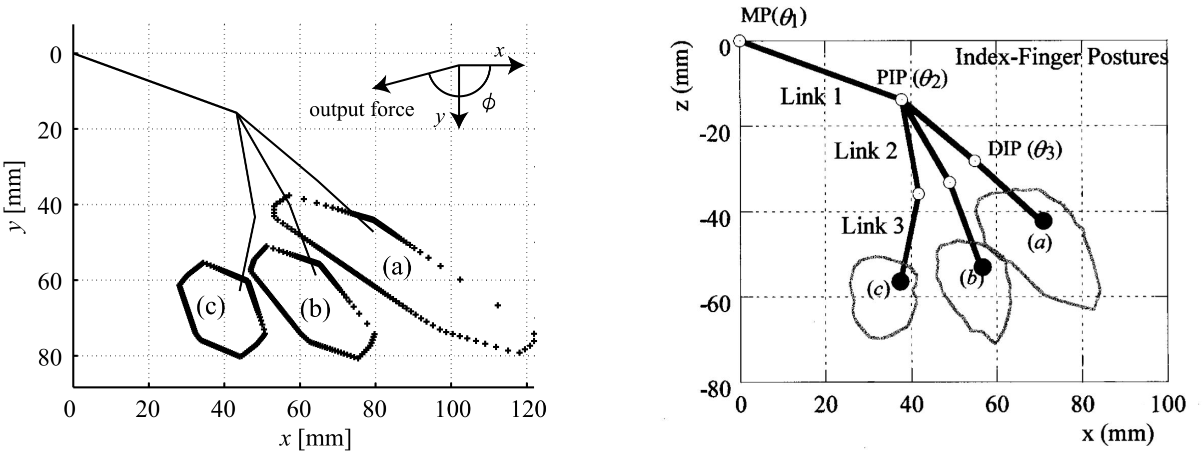
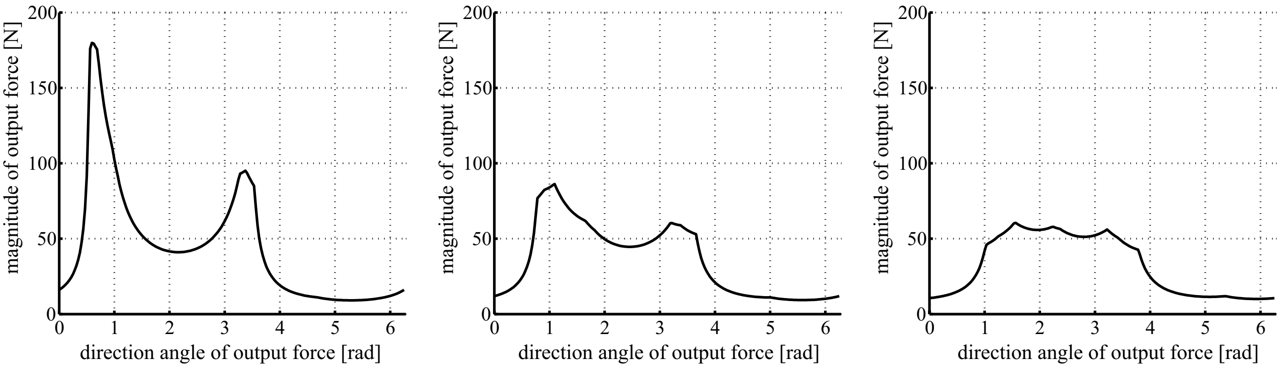


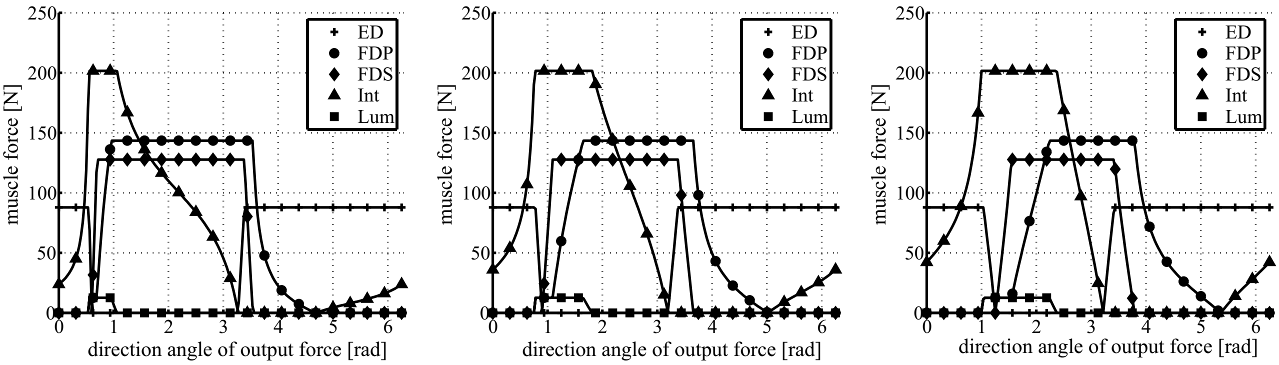
(1) Calculated maximum output force distribution. (2) Measured maximum output force distribution[8].

Fig. 3 Maximum output force distribution at case (a), (b) and (c).



(1) case (a) (2) case (b) (3) case (c)

Fig. 4 Magnitudes of maximum output force at case (a), (b) and (c).



(1) case (a) (2) case (b) (3) case (c)

Fig. 5 Muscle forces at case (a), (b) and (c).

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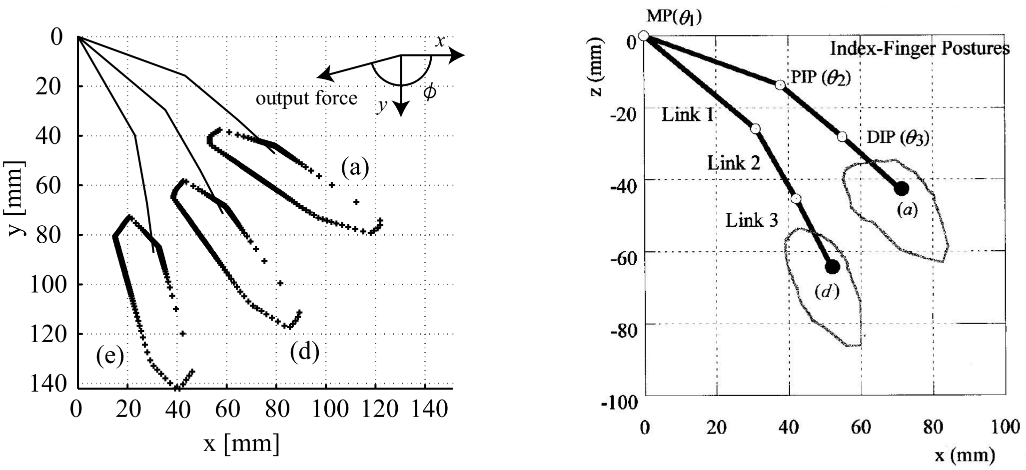
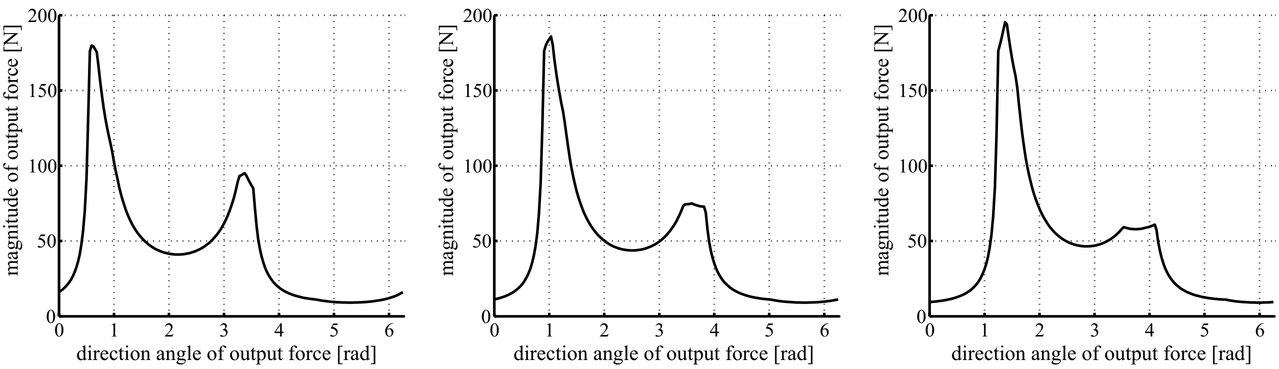


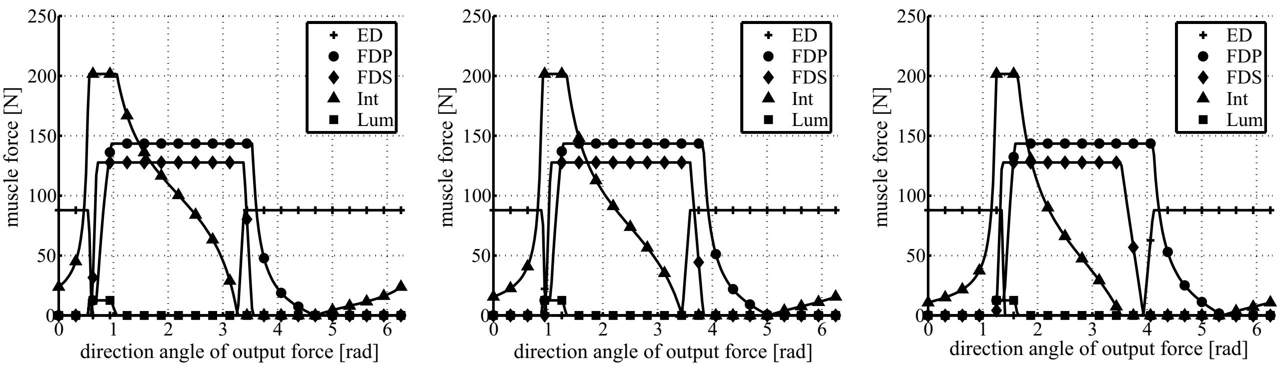
(1) Calculated maximum output force distribution. (2) Measured maximum output force distribution[8].

Fig. 6 Maximum output force distribution at case (a), (d) and (e).



(1) case (a) (2) case (d) (3) case (e)

Fig. 7 Magnitudes of maximum output force at case (a), (d) and (e).



(1) case (a) (2) case (d) (3) case (e)

Fig. 8 Muscle forces at case (a), (d) and (e).