Control of a Straight Line Motion for a Humanoid Robot using Characteristics of Bi-Articular Simultaneous Drive and Machine Learning Control

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1. *Introduction*

Technologies for a humanoid robot are advancing and a lot of kinds of biped robots have being designed these days. From the point of its motion control, consideration of bi-articular simultaneous drive is being popular now.

Each joint torque is controlled individually for conventional humanoid by using complicated kinematic calculations such as real-time inverse kinematics. On the other hand, it has been already discussed by some researchers that biological objects like a human or an animal do not always perform the motion control based on complex calculations but utilize the features of the mechanisms. One of the characteristic mechanisms is cooperation of mono-articular drive and bi-articular simustaneous drive. Mono-articular muscle connects between one joint and one link and it makes rotational contractive force to the joint. In addition, bi-articular muscle connects between two adjacent joints and it generates the contractive force to those joints simultaneously. There is a multi-articular muscle for some animals, but cooperation of mono-articular and bi-articular simultaneous drive of a human being is especially treated in this paper.

The authors have considered the characteristics of dynamic motion of the humanoid robot from mathematical and engineering point of view. In this paper, force characteristics at the tip point using cooperation of mono-articular and bi-articular simultaneous drive from previous research [1] is mathematically derived in Section 2. Moreover, the authors focus on the straight line motion from the supporting point, and the characteristic of each joint torque and force at the tip point is mathematically discussed in Section 3 and 4. Finally, designing of a motion control based on statistic characteristics in Section 2 and compensation of non-linear parts during the dynamic motion is summarized.

2. *Actuator control using cooperation of bi-articular simultaneous drive mechanism on static condition*

2.1. *Mathematical calculation of force characteristics on static condition*

Characteristic of output force at the tip point depending on joint torques which is controlled by mono-articular and bi-articular simultaneous drives is mathematically discussed in this section.

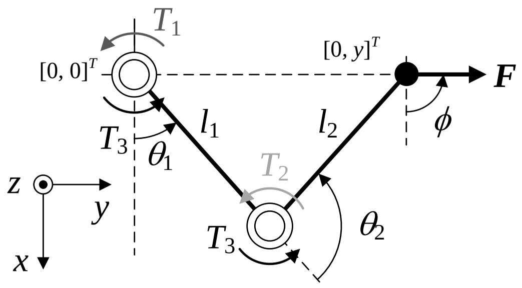

Fig. 1 Fundamental arm model.

Fig. 1 shows calculation model of 2-link arm system. When shoulder joint is assumed as supporting point, tip position $\boldsymbol{x}$ in Fig. 1 is described as (1).

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{bmatrix} \quad (1)$$

Where, l_1, l_2 : Length of each link (m),
θ_1, θ_2 : Angle of each joint (rad)

Jacobian matrix $\boldsymbol{J(\theta)}$ at the tip point is calculated by the differential of angle;

$$\boldsymbol{J}(\boldsymbol{\theta}) = \begin{bmatrix} -l_1 \sin\theta_1 - l_1 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix} \quad (2)$$
$$= \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

There is a relationship between joint torques $\boldsymbol{\tau}$ and force at the tip point $\boldsymbol{F}$ from a principle of virtual work;

$$\boldsymbol{\tau} = \boldsymbol{J}^{\mathrm{T}}(\theta)\boldsymbol{F} \quad (3)$$

Here, concept of actuator torques which are consisted of mono-articular muscle and bi-ariticular muscle is explained. Two-link upper arm model of a human is shown in Fig. 2. There are six muscle components in the figure, and e3 and f3 are the bi-articular muscles. As described in (3), torques for the motion of a humanoid robot is considered at each joint point as τ_1, τ_2. However, two adjacent joints are received rotational torques simultaneously when the bi-articular muscles generate contractive force. Rotational torques are derived from radii of joints multiply output forces by muscles contractions respectively. Therefore, each joint torque in Fig. 1 is calculated as following equation [3];

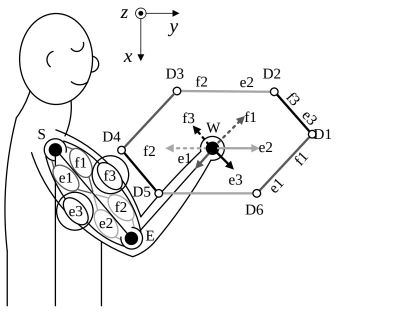

Fig. 2 Muscle components of an upper arm of a human.

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}r_1(F_{f1}+F_{e1})+r_1(F_{f3}+F_{e3})\\ r_2(F_{f2}+F_{e2})+r_2(F_{f3}+F_{e3})\end{bmatrix}=\begin{bmatrix}T_1+T_3\\ T_2+T_3\end{bmatrix} \quad (4)$$

If we consider that length of each link ($l_1=l_2=l$) and radius of each joint ($r_1=r_2=r$) in Fig. 1 are same values respectively, force at the tip point $\boldsymbol{F}$ can be described as (5) by using actuator torques $\boldsymbol{T}$. Furthermore, force at the tip point $\boldsymbol{F}$ is expressed by the amplitude F and direction ϕ as illustrated in Fig. 1. Also, trigonometrical function is expressed as following values in this paper; where $\cos\theta_1 = \mathrm{C}_1$, $\sin(\theta_1+\theta_2) = \mathrm{S}_{12}$ [4];

$$\begin{bmatrix}FC_\phi\\ FS_\phi\end{bmatrix}=\frac{1}{l^2\mathrm{S}_2}\begin{bmatrix}l\mathrm{C}_{12} & -l(\mathrm{C}_1+\mathrm{C}_{12})\\ l\mathrm{S}_{12} & -l(\mathrm{S}_1+\mathrm{S}_{12})\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\frac{1}{l\mathrm{S}_2}\begin{bmatrix}\mathrm{C}_{12}T_1-(\mathrm{C}_1+\mathrm{C}_{12})T_2-\mathrm{C}_1T_3\\ \mathrm{S}_{12}T_1-(\mathrm{S}_1+\mathrm{S}_{12})T_2-\mathrm{S}_1T_3\end{bmatrix} \quad (5)$$

2.2. *Designing of each actuator torque using cooperation of mono-articular and bi-articular simultaneous drive depending on the force at the tip point*

Relationship between the force at the tip point and sum of each actuator torque was mathematically explained at previous section. Actual designing of each actuator torque based on real muscle responds of a human is discussed in this section. As it has explained in an introduction, we have to calculate each joint torque using real-time calculation of (3), when there is some external force $\boldsymbol{F}$ at the tip point.

On the other hand, Kumamoto *et al*., have analyzed and measured characteristics and responds of a human muscle using Electromiogram (EMG). They had an experiment by following way. An arm of a human subject fixed at a specific position as illustrated in Fig. 1. Then, the subjects generated a force from a wrist point toward six-directions while the wrist was fixed at the position. Six-directions mean positive and negative ones parallel to the line connecting the shoulder and the wrist, parallel to the line connecting the shoulder and the elbow, to the line connecting the elbow and the wrist respectively. These directions show a basis of evaluation of the direction of the force. The force was measured by strain gauges. In addition, the actuating stimulus to each muscle in the moment was measured by EMG [1] [2].

Fig. 3 shows a result of EMG. It has been cleared from a figure that direction of the force at the tip point ϕ is controlled by sum of each muscle force and control of the force is made of case patterns of linear functions simply. This is due to bi-articular simultaneous drive and it is impossible to use the pattern for conventional individual each joint torque control.

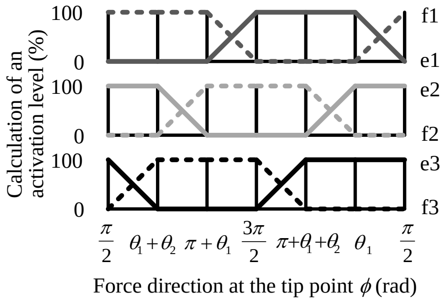

Fig. 3 Each muscle respond depending on the direction of the force at the tip point [1].

One typical example is explained as follow. In case of that parameter ϕ is $\pi/2$ and the position of the arm is on the y-axis as in Fig. 1, muscle f1, e2 and e3 are generating maximal contractive force respectively. There is a relationship between each joint angle when length of each link is same value;

$$2\theta_1+\theta_2=\pi,\ \ \mathrm{C}_1+\mathrm{C}_{12}=0\ \text{ and }\ \mathrm{S}_1=\mathrm{S}_{12} \quad (6)$$

In this situation, x-direction force becomes zero and y-direction force is depending on the shoulder joint angle θ_1.

We can consider three cases by the direction ϕ. The clockwise rotational direction is defined as a positive, *i.e*., the extensor muscles generate positive rotational torques and the flexor muscles generate negative torques respectively. Also, it is assumed that maximal value of each muscle is same.

Case 1 : $\frac{3}{2}\pi \le \phi < \pi+\theta_1+\theta_2,\ \frac{1}{2}\pi \le \phi < \theta_1+\theta_2$

It can be considered from the EMG result that, amplitude of T_1 and T_2 is same and opposite values. Each actuator torque is derived from (3);

$$\begin{bmatrix}FC_\phi\\ FS_\phi\end{bmatrix}=\begin{bmatrix}-\frac{1}{2l\mathrm{S}_1}(T_1+T_3)\\ \frac{1}{2l\mathrm{C}_1}(T_1+2T_1-T_3)\end{bmatrix} \quad (7)$$

$$T_1=\frac{lF}{2}(-\mathrm{S}_1\mathrm{C}_\phi+\mathrm{C}_1\mathrm{S}_\phi),\ \ T_2=-T_1\ \text{ and }\ T_3=-\frac{lF}{2}(3\mathrm{S}_1\mathrm{C}_\phi+\mathrm{C}_1\mathrm{S}_\phi) \quad (8)$$

In the same way, each actuator torque is able to design using EMG result for another two cases [4].

Case 2 : $\pi+\theta_1 \le \phi < \frac{3}{2}\pi,\ \theta_1 \le \phi < \frac{1}{2}\pi$

$$\begin{bmatrix}FC_\phi\\ FS_\phi\end{bmatrix}=\begin{bmatrix}-\frac{1}{2l\mathrm{S}_1}(T_1+T_3)\\ \frac{1}{2l\mathrm{C}_1}(T_1-2T_3-T_3)\end{bmatrix} \quad (9)$$

$$T_1=\frac{lF}{2}(-3\mathrm{S}_1\mathrm{C}_\phi+\mathrm{C}_1\mathrm{S}_\phi),\ \ T_3=-\frac{lF}{2}(\mathrm{S}_1\mathrm{C}_\phi+\mathrm{C}_1\mathrm{S}_\phi)\ \text{ and }\ T_2=T_3 \quad (10)$$

Case 3 : $\pi+\theta_1+\theta_2 \le \phi < 2\pi$ and $0 \le \phi < \theta_1$, $\theta_1+\theta_2 \le \phi < \pi+\theta_1$

$$\begin{bmatrix}FC_\phi\\ FS_\phi\end{bmatrix}=\begin{bmatrix}-\frac{1}{2l\mathrm{S}_1}(T_1+T_3)\\ \frac{1}{2l\mathrm{C}_1}(-2T_2)\end{bmatrix} \quad (11)$$

$$T_1=-lF\mathrm{S}_1\mathrm{C}_\phi,\ \ T_3=T_1\ \text{ and }\ T_2=-lF\mathrm{C}_1\mathrm{S}_\phi \quad (12)$$

Calculation costs of joint torque control using the case patterns and conventional calculation using Jacobian matrix are same on the static condition, however the authors consider that the proposed control utilize the designing of joint torques during the dynamic motion. Characteristics of dynamic motion are taken into account in the following section.

3. Characterization of dynamic straight line motion

3.1. *Motion equations of two-link arm for straight line*

In this section, characteristic of dynamic motion of the two-link arm model is discussed. Mathematical calculation of the motion is based on Lagrange equation of motion in this paper. Each joint torque is calculated by conventional method, therefore only some important equations are described and explained in this paper.

Fig. 4 shows two-link arm model and straight line motion from supporting point to *z*-axis direction is considered. In order to consider gravity acceleration during the motion, coordinate of the tip position is changed from $[x, y]^T$ to $[x, z]^T$ from this section. Moreover, parameters of components of the arm are shown as TABLE I. Although calculation of joint torques is described using general parameters as shown in TABLE I, it is assumed that length of each arm, radius of each joint and mass of each link are same values respectively for the model. In addition, length between joint *i* and centre of gravity of link *i* is set to $l_{g1} = l_{g2} = l/2$. Mass of load at the tip point is also assumed as m_L. Moreover, length of load is defined as l_L, and length between the tip and centre of gravity of load is set to $l_{gL} = l_L/2$. Radius of rotation of the load is also assumed as same as the link 2.

Cylindrical shapes are assumed for each link and the load in this paper. Therefore, moments of inertia of links and the load are described as follows;

$$I_1 = I_2 = m(\frac{r^2}{4} + \frac{l^2}{12}), \quad I_L = m_L(\frac{r^2}{4} + \frac{l_L^2}{12}) \qquad (13)$$

Furthermore, length between joint 2 and centre of gravity of link 2 with load is described as follow;

$$l_{g2L} = l_{g2} + \frac{m_L}{m_2 + m_L}(l_{g2} + l_{gL}) \qquad (14)$$

Consequently, moment of inertia of link 2 with load can be derived;

$$I_{2L} = I_2 + m_2\left(\frac{m_L}{m_2 + m_L}(l_{g2} + l_{gL})\right)^2 + I_L + m_L\left(\frac{m_2}{m_2 + m_L}(l_{g2} + l_{gL})\right)^2 \qquad (15)$$

Equation of motion is generally derived as (16);

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}) \qquad (16)$$

$$\boldsymbol{M}(\boldsymbol{\theta}) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \qquad (17)$$

$\boldsymbol{M}(\boldsymbol{\theta})$ means an inertia matrix, $\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ means centrifugal and Coriolis forces, and also $\boldsymbol{g}(\boldsymbol{\theta})$ means gravity in the equation.

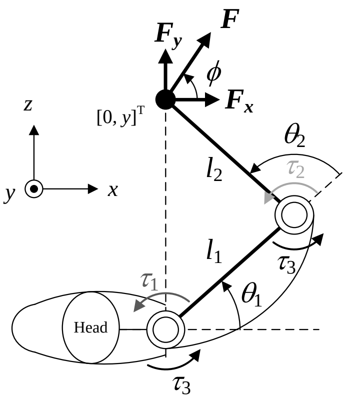

Fig. 4 Two-link arm model for dynamic vertical direction motion.

TABLE I

Parameters of a two-link robot arm.

Symbol	Definition of the parameter
m_i	Mass of link *i* (kg)
l_i	Length of link *i* (m)
l_{gi}	Length between joint *i* and centre of gravity of link *i* (m)
I_i	Moment of inertia through the centre of gravity of link *i* (kgm^2)
τ_i	Rotational torque of each joint (τ_1, τ_2) (Nm)
T_i	Each actuator torque (T_1, T_2, T_3) (Nm)
g	Acceleration of gravity = 9.8 (m/sec^2) (-*z* - direction)

Terms in (17) are general ones and the authors have already written the details in [4]. Hence, angular accelerations of shoulder and elbow joints can be calculated by inverse matrix of inertia matrix and so on.

3.2. *Characteristics of an acceleration at the tip point*

Characteristic of an acceleration of the tip point is mathematically explained in this section. The two-dimensional tip acceleration in Fig. 4 is described using angular acceleration of each joint and time-derivative of the tip position;

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{M}^{-1}(\boldsymbol{\theta})\left(\boldsymbol{\tau} - \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) - \boldsymbol{g}(\boldsymbol{\theta})\right) + \dot{\boldsymbol{J}}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} J_{11}M_{22} - J_{12}M_{21} & -J_{11}M_{12} + J_{12}M_{11} \\ J_{21}M_{22} - J_{22}M_{21} & -J_{21}M_{12} + J_{22}M_{11} \end{bmatrix}\begin{bmatrix} \tau_{hg1} \\ \tau_{hg2} \end{bmatrix} - \begin{bmatrix} (l_1C_1 + l_2C_{12})\dot{\theta}_1^2 + l_2C_{12}(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \\ (l_1S_1 + l_2S_{12})\dot{\theta}_1^2 + l_2S_{12}(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \end{bmatrix} \qquad (18)$$

Where, $\Delta = M_{11}M_{22} - M_{12}M_{21}$: determinant of the inertia matrix. In (18), terms of joint torques, centrifugal forces and Coriolis forces and also gravity factors are shown as follows respectively [4];

$$\tau_{hg1} = \tau_1 + (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) - g\left\{m_1 l_{g1}\mathrm{C}_1 + (m_2 + m_L)(l_1\mathrm{C}_1 + l_{g2L}\mathrm{C}_{12})\right\}$$

$$\tau_{hg2} = \tau_2 - (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_1^2 - g(m_2 + m_L)l_{g2L}\mathrm{C}_{12} \quad (19)$$

Hence, characteristic of tip acceleration can be derived generally using (18) - (19) and straight line motion is discussed in the next section.

3.3. *Characteristics joint torques for straight line motion of the tip on the z-axis*

Specific motion is taken into account as same as Section 2. In this paper, straight line motion as illustrated in Fig. 4 is considered for characterization of the cooperation of mono-articular and bi-articular simultaneous drive. Because a human often uses bi-articular muscles for stretching straightly motion like pushing a wall, squatting or walking.

Moreover, it is assumed that lengths of links, radii of joins and masses of links are same values respectively. The authors have already considered that same lengths of links and same radii of joints make easy calculation for designing the direction of external force at the tip point. In addition, calculation of centre of gravity of each value becomes easy if we assume that masses of links are same values.

Acceleration of x-direction of the tip which is described in (18) should be kept zero and velocity of x-direction of the tip of the tip should be kept as well under the straight line motion. They can be calculated as follows respectively based on the assumption;

$$\dot{x} = l_1\mathrm{S}_1\dot{\theta}_1 - l_2\mathrm{S}_{12}(\dot{\theta}_1 + \dot{\theta}_2) = 0 \quad \therefore 2\dot{\theta}_1 + \dot{\theta}_2 = 0$$

$$\ddot{x} = 0 \quad \therefore \frac{\tau_{hg2}}{\tau_{hg1}} = \frac{J_{11}M_{22} - J_{12}M_{21}}{J_{11}M_{12} - J_{12}M_{11}} \quad (20)$$

Finally, each joint torque during straight line motion can be derived using (18) – (20).

$$\tau_1 = \frac{1}{2}\left\{-I_1 - m_1 l_{g1}^2 + I_2 + m_2(l_{g2}^2 - l_1^2) + I_L + m_L(4l_{g2}^2 + 4l_{g2}l_{gL} + l_{gL}^2 - l_1^2)\right\}\ddot{\theta}_2 + g\left\{m_1 l_{g1}\mathrm{C}_1 + (m_2 + m_L)(l_1\mathrm{C}_1 + l_{g2L}\mathrm{C}_{12})\right\} \quad (21)$$

$$\tau_2 = \frac{1}{2}\left\{I_2 + m_2(l_{g2}^2 - l_1 l_{g2}\mathrm{C}_2) + I_L + m_L(4l_{g2}^2 + 4l_{g2}l_{gL} + l_{gL}^2 - 2l_1 l_{g2}\mathrm{C}_2 - l_1 l_{gL}\mathrm{C}_2)\right\}\ddot{\theta}_2 + (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_1^2 + g(m_2 + m_L)l_{g2L}\mathrm{C}_{12}$$

It is obvious from (21) that there is no nonlinear term for joint torque of shoulder point τ_1 without gravity acceleration condition *i.e.*, horizontal line motion. It means that designing of shoulder joint torque is depending on constant values which are from characteristics of links, masses etc. and variable number of angular acceleration of elbow. Moreover, the authors have been considered that the term of gravity factor is not large one for the calculation cost of the control. Hence, designing of shoulder joint torque is totally not so difficult for specific straight line motion.

On the other hand, elbow joint torque τ_2 has nonlinear term due to cosine function of elbow joint angle θ_2, therefore discussion for the designing of elbow joint torque is needed.

4. *Designing of joint torque control based on the loading motion characteristics*

4.1. *Definition of joint torque under dynamic motion with consideration of d'Alembert's principle*

It is able to separately consider that joint torques which are for the links themselves and for the external force at the tip point. In general, d'Alembert's principle is used for the expansion of principle of virtual work and it can be applied for dynamic calculation.

First, d'Alembert's principle is simply explained. Sum of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system, is zero;

$$\boldsymbol{F} - m\frac{d^2\boldsymbol{r}}{dt^2} = 0 \quad (22)$$

The total virtual work of the impressed forces plus the inertial forces plus the inertial forces vanishes for reversible displacements. This shows that the expansion of a principle of virtual work is possible to dynamic condition by the principle [5].

Consequently, each joint torque for the straight line motion is derived as following equation from (5) and (21) [4].

$$\begin{bmatrix}\tau_{1\mathrm{Total}} \\ \tau_{2\mathrm{Total}}\end{bmatrix} = \mathrm{eq.}(21) + \begin{bmatrix}-2l\mathrm{S}_{12} & 0 \\ -l\mathrm{S}_{12} & l\mathrm{C}_{12}\end{bmatrix}\begin{bmatrix}F_x \\ F_z\end{bmatrix} \quad (23)$$

It is obvious from (23) that there is no rotational torque for shoulder joint depending on the load, when $F_x = 0$ and the tip of the arm is kept on the z-axis. $\tau_{1\mathrm{Total}}$ is perfectly designed by (21) on the condition. Hence, designing of $\tau_{2\mathrm{Total}}$ is taken into account at the next session.

4.2. *Characterization of elbow joint torque*

It is assumed in this paper that external force at the tip point is mainly worked for a load factor in the system. Consequently, some factors depending on the parameter m_L, l_L and I_L in (21) are assumed as zero and a load which has no rotation during the motion is considered in stead of them.

General designing of elbow joint torque is completely real-time calculation using the equation (21). However, it is actually a difficult calculation because of nonlinear factors depending on cosine function of joint angle. On the other hand, some biological researchers have said that a human realize some easy motion using bi-articular simultaneous drive as mentioned in the introduction. There are a lot of publications which are focused on responds of some muscles during dynamic motion like walking, jumping, squatting etc. , and those locomotion have been considered by using Electromiogram as same as Kumamoto's group [1] [2].

However, it is difficult for engineers that measurement result of EMG is only amplitude of each muscle and there is no real information about muscle force. Therefore, the authors have focused on the factors of elbow joint torque for links themselves τ_2 and for the external force at the tip T_2+T_3. TABLE II shows the parameters for numerical analysis.

TABLE II

Parameters of the two-link arm and a load.

Definition of each parameter	Value
Radius of rotation of each arm : r (r_1, r_2)	0.0300 (m)
Length of each link : l (l_1, l_2)	0.300 (m)
Mass of each link : m (m_1, m_2)	2.00 (kg)
Moment of inertia of each arm : I (I_1, I_2)	0.0155 (kgm^2)
Initial position of the tip : (x, z)	$[0,0]^T$ (m)
Mass of load : m_e	1.00 (kg)
Amplitude of the acceleration of motion : α	1.00 (m/sec^2)

Mass of load is defined as m_e instead of m_L in (21). Value of m_e is firstly defined as 1.00kg due to the values of mases of links and it is assumed that the load has no rotational motion as explained above in this paper.

Equation of motion of the load is described as (24); where α means amplitude of an acceleration of the load and the tip, ψ means two-dimensional direction of the acceleration respectively. ψ is set to $\pi/2$ during the straight line motion;

$$\boldsymbol{F_e} = \begin{bmatrix} F_x \\ F_z \end{bmatrix} = m_e \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = m_e \begin{bmatrix} \alpha C_\psi \\ \alpha S_\psi + g \end{bmatrix} \tag{24}$$

Therefore, direction of the external force ϕ becomes $\pi/2$ during the motion.

Joint torques for the external force are expressed using sum of actuator torques in (5) as explained in Section 2 in order to proposed the advantage of bi-articular simultaneous drive;

$$\begin{bmatrix} T_1 + T_3 \\ T_2 + T_3 \end{bmatrix} = \boldsymbol{J}^T(\boldsymbol{\theta}) \begin{bmatrix} F_x \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ m_e l C_{12}(\alpha + g) \end{bmatrix} \tag{25}$$

Also, angular velocities are calculated by relationship of Jacobi matrix and tip velocity, and angular accelerations are calculated as explained before;

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}(\boldsymbol{\theta})\dot{\boldsymbol{x}}, \quad \ddot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}(\boldsymbol{\theta})(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}) \tag{26}$$

Consequently, elbow joint torque for links themselves τ_2 can be calculated from (21) and (26);

$$\tau_2 = \frac{1}{2}\left\{I_2 + m_2(l_g^2 - l l_g C_2)\right\}\left(-\frac{S_1 + S_{12}}{l S_2}\ddot{z} - \frac{S_1 S_{12}(S_1 + S_{12})^2}{l^2 S_2^3}\dot{z}^2\right) + \frac{m_2 l_g S_{12}}{l S_2}\dot{z}^2 + g m_2 l_g C_{12} \tag{27}$$

Finally, the authors have considered that designing of elbow joint torque τ_{2Total} is possible to use only the characteristic of the torque for the external force T_2+T_3 [6].

4.3. *Numerical analysis of joint torques*

In this section, characterizations of (25) and (27) are mathematically discussed and designing of elbow joint torque based on T_2+T_3 is taken in to account.

Trajectory of the tip point is shown in Fig. 5. Motion time has been set as 1sec and interval of calculation has been set as 0.01sec. Cosine function of time has been used for the amplitude of the acceleration α and the direction ψ has been kept to $\pi/2$rad for the straight line motion.

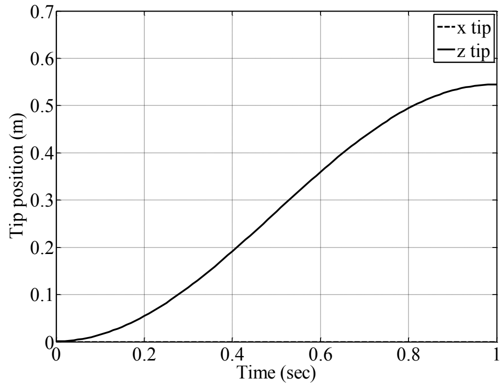

Fig. 5 Trajectory of the arm $\boldsymbol{x}$ (m) : x-tip means tip position of x-direction, z-tip means tip position of z-direction.

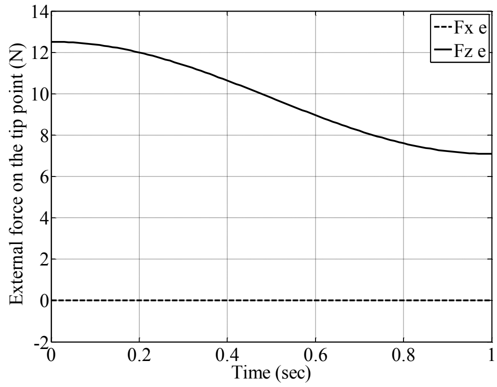

Fig. 6 Time alternation of the external force $\boldsymbol{F_e}$ at the tip (N) : Fxe means x-direction force, Fze means z-direction force.

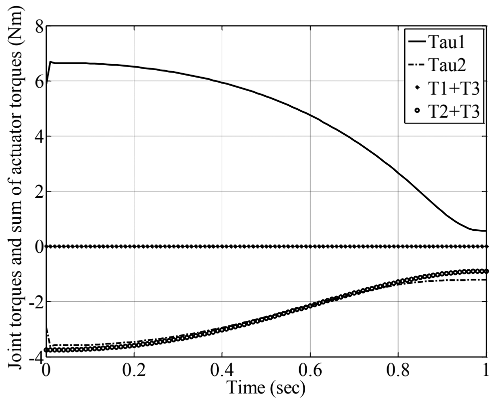

Fig. 7 Time alternation of terms of each joint torques τ_1, τ_2, T_1+T_3 and T_2+T_3 (Nm) : Tau1 and Tau2 mean torques for links, T_1+T_3 and T_2+T_3 mean torques for a load at the tip point.

Time alternation of external force $\boldsymbol{F_e}$ is shown in Fig. 6, which is based on (25) and ϕ is always $\pi/2$rad. In the figure, T_1+T_3 was always zero during the motion because there is no x-direction force for the load during the motion.

Fig. 7 shows a simulation result of time alternation of factors of joint torques. In the figure, τ_1 and τ_2 mean rotational torques which are calculated from (21) and T_1+T_3 and T_2+T_3 mean torques which are calculated from (25) respectively.

The simulation was based on the parameters of TABLE II

and the amplitude α. As we can see from the figure, τ_2 is almost close to T_2+T_3 during the motion. Consequently, the authors have considered and proposed that designing of elbow joint torques τ_{2Total} is realized only using the T_2+T_3.
It means that τ_{2Total} is designed as T_2+T_3 at no load condition or τ_{2Total} is designed as $2(T_2+T_3)$ when the mass of load m_e is 1kg. Because, characteristic of elbow joint torque for links themselves τ_2 is approximation to the characteristic of the torque which is needed for the 1kg mass's load during the straight line motion. If it is possible to assume as the motion of one mass system, we do not any complicated real time calculation for a robot [6].

Higher acceleration of the tip and load has been simulated as the other case. Fig. 8 shows the trajectory of the arm. Motion time was changed from Fig. 5. The motion has been fixed without consideration of singular point *i.e.*, the simulation has not just used 10 times value of α compare to the previous one. Fig 9 shows time alternation of the external force $\boldsymbol{F_e}$ and Fig. 10 shows time alternation of factors of joint torques respectively. Mass of the load m_e has been changed from 1kg to 0.8kg in order to adjust the total torques of τ_2 and T_2+T_3.

At first, we can see much higher values of forces and torques compare to the previous simulation. In fact, designing the locomotion is not suitable for a humanoid robot which the authors target on the research subject. Also it can be considered that this kind of very quick motion is difficult for human, animals and so on. However, designing of elbow joint torque might be able to apply by changing of the value of m_e. For example, a human is not able to catch a heavy ball when it has high speed. On the other hand, it might be able to catch if the ball is right weight. If it can be assumed that joint torque for speedy motion of the arm is designed by an image of some right weight load, the suggestion would be utilize.

4.4. *Consideration of utility of bi-ariticular simuletaneous drive*

If designing of elbow joint torque during dynamic motion is close to static conditions, cooperation of mono-articular and bi-articular simultaneous drive would be apply to the torque control. When the control will be realized using the result of EMG as explained in Section 2, informations of the external force at the tip point, joint angles and angular velocities are very important. This consideration is actually designing for elbow joint torques τ_{2Total} but it will be also useful for the compensation of gravity factors of shoulder joint angle.

However, it is obvious from Fig. 7 and Fig. 10 that elbow joint torque for links themselves τ_2 is not completely correspond to the torque T_2+T_3. In order to compensate the control of total elbow joint torque τ_{2Total}, case patterns control using the direction of the external force ϕ will be additionally applied. Control of the direction ϕ is easily able to realize by cooperation of mono-articular and bi-articular simultaneous drive. When a two-link humanoid robot have a force sensor in order to know amplitude of force of each axis and encoder for each joint, real-time calculation of motion control will be easily designed compare to conventional real-time calculation using Lagrange equation of motion.

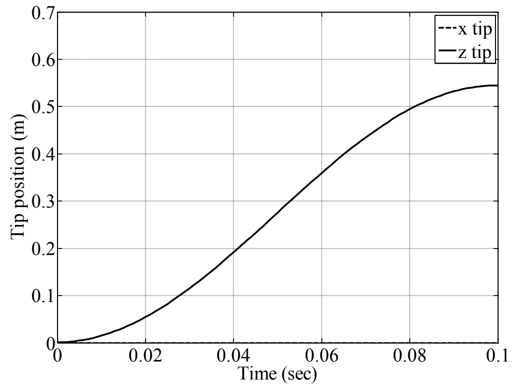

Fig. 8 Trajectory of the arm $\boldsymbol{x}$ (m), fast motion : x-tip means tip position of *x*-direction, z-tip means tip position of *z*-direction.

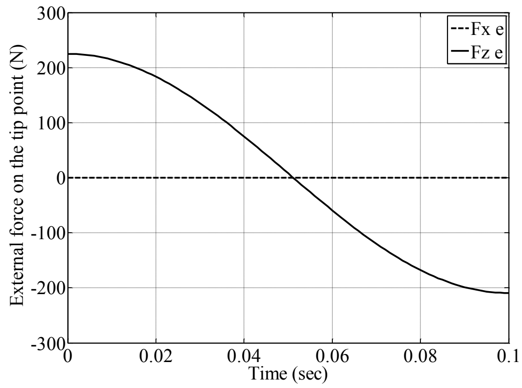

Fig. 9 Time alternation of the external force $\boldsymbol{F_e}$ at the tip (N), fast motion : Fxe means *x*-direction force, Fze means *z*-direction force.

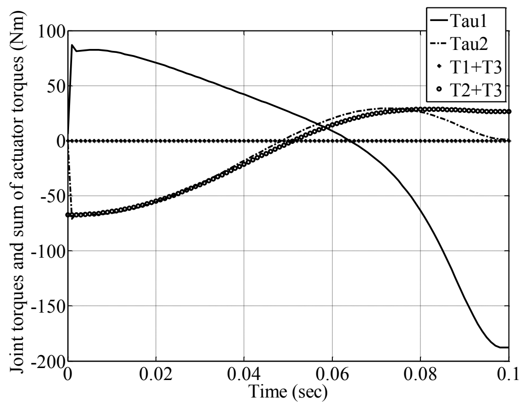

Fig. 10 Time alternation of terms of each joint torques τ_1, τ_2, T_1+T_3 and T_2+T_3 (Nm) , fast motion : Tau1 and Tau2 mean torques for links, T_1+T_3 and T_2+T_3 mean torques for a load at the tip point.

Furthermore, the authors are considering using machine learning control for no load condition at the tip point. It is impossible to control using the force sensor information without load condition. Feedback controller which is explained in Section 2 does not perform at no load condition, therefore there is no compensation for the error between elbow joint torque which is needed for the straight line motion and the proposed torque which is designed using

T_2+T_3. If a robot has a machine learning control system, the condition and designing of shoulder joint torque with gravity factors in (21) would be completely controlled.

5. *Conclusion*

In order to realize an easy motion control of humanoid robot limbs, control using cooperation of mono-articular and bi-articular simultaneous drive of biological characteristics is proposing from some researchers. In this paper, designing of sum of actuator torques which are roles of mono-articular and bi-ariticular simultaneous drives has been mathematically calculated and evaluated from the point of biological characteristics. The characteristics means the measurement result of each muscle respond using Electromiogram (EMG) and force characteristics at the tip point. Amplitude and direction of the force depending on each actuator torque has been completely calculated.

Furthermore, characteristics of joint torques during straight line dynamic motion have been also mathematically considered. It has been cleared that shoulder joint torque can be designed by constant values of links characteristics and angular acceleration of elbow and gravity factors. The constant values are realized only for straight line motion, *i.e*,. the authors have considered that bi-articular simultaneous drive is especially useful for straight line motion.

In addition, the authors have considered that elbow joint torque for links themselves is approximation to the torque which is needed for the external force at the tip point. When the straight line motion is operated at normal speed like a human or an animal are doing, designing of elbow joint torque for links themselves becomes approximately same to one mass problem.

Consequently, the authors have proposed that designing of elbow joint torque is basically used of the control which is for the external force for a load at the tip point. In addition, the compensation of error between actual elbow joint torque which is needed for straight line motion and proposed torque which is designed for the external force and its expansion is realized using sum of actuator torque control depending on the direction of the external force. Also, the authors have considered that combined control using machine learning control which is especially for the compensation of no load condition at the tip point and the compensation of gravity factor for shoulder joint torque.

As a future work, evaluation of the proposed control using experimental machine will be needed. The authors are designing the machine which has electrical and gearless motors, rotary encoders and force sensors. Finally, verification of motion characteristics using bi-articular simultaneous drive will be taken into account. Also, combined torque control using real-time case patterns and machine learning control for the compensation of the proposed torque control will be concretely considered.

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