Control of a Straight Line Motion for a Two-Link Robot Arm Using Coordinate Transform of Bi-articular Simultaneous Drive

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***Abstract*—There are some difference between biological subjects and conventional humanoid robots from the point of its mechanisms, control method, and application of the biological characteristics to humanoid robots are now being popular.**

Motion control for 2-link robot arm on specific condition are mainly discussed in this paper. From the point of mechanism, biological subjects have two joints simultaneous drive. Though benefits and characteristics of the mechanism on static condition have already cleared from the measurement of previous research, it has been calculated and discussed mathematically in this paper. In addition, dynamic motion control of a specific condition has been also calculated and considered. Furthermore, designing of each actuator torque which is based on the Electromiogram result and its control for external force on the tip point is concretely described. Finally, those characteristics of static and dynamic motions with bi-articular simultaneous drive on specific condition are summarized.

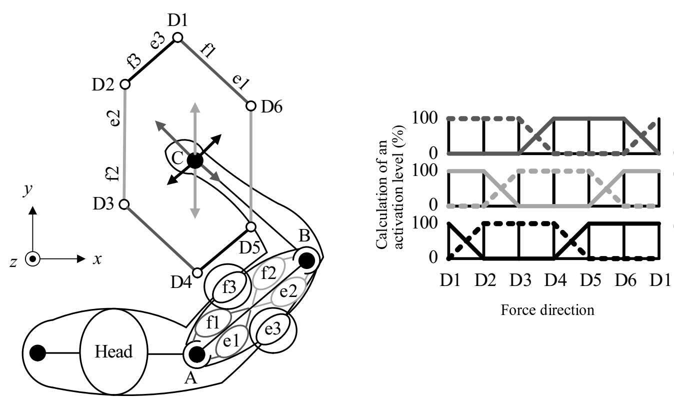

Figure 1. Force direction from EMG measurement on static condition.

I. Introduction

Technologies for humanoid robots are greatly advancing these days. Motion control is one of the essential parts in many kinds of technologies and applications for the humanoid robot. In fact, motion control technology of a humanoid robot like, *e.g.*, ASHIMO of HONDA Co. Ltd. has being advanced by a mechanical model which can be realized complicated kinematic and dynamic calculations and by a high performance processor for the calculation.

However, there are some differences between these conventional humanoid robots and biological objects from the point of mechanism and its control. Some researchers who study biology and medical science consider and indicate that biological objects do not always perform a motion control by complex calculation but utilize the features of the mechanism normally[1][2]. When we consider the mechanism of joint control, joints of biological objects are composed of bones, ligaments and muscles mainly, and it seems that there are no position sensor for measuring angles of joints.

When we focus on the muscle composition of human beings which connects a joint and a link, we have one characteristic muscle, which is called a bi-articular muscle. Figure 1 shows the top view of an arm model of human beings. We can see a mono-articular muscle which connects between one joint and one link from the left parts of the figure. Moreover, human beings has bi-articular muscles like e3 and f3 as illustrated in Figure 1. One of the characteristic of a bi-articular muscle is that it connects between two joints and the bi-articular muscles generate the contractive force to those joints simultaneously. All quadruped and biped animals have the bi-articular muscles, and it is said that standard motions using the bi-articular muscles are realized without any complicated dynamic calculations.

On the other hand, the simultaneous actuation of two joints by a bi-articular muscle was not considered in conventional designs of artificial humanoids. They have, for instance, rotational motors at their cubital and shoulder joints, which operate individually as the role of mono-articular muscles. The structure often requested complicated real-time inverse-kinematic calculations for the dynamic motion control. The principle of inverse kinematics has been already realized and motion control by using the principle and angular information of each joint has been already applied a lot of humanoid robots as well, but it is not so easy to solve the calculation quickly because it has always nonlinear parts.

In this paper, possible advantages of the bi-articular simultaneous drive are summarized from engineering point of view from previous research[1]. Dynamic motion in case the tip of the arm moves straightly is mathematically analyzed for explaining substantial engineering advantages of considering a bi-articular muscle in the robot arm in Sections II-IV. Furthermore, control design using the advantages are discussed in Section V.

II. Characteristics of Force on the tip point with consideration of bi-articular simultaneous drive

A. Characteristics of force control cooperated with bi-articular muscles

First, force characteristic on the tip point with bi-articular muscle is explained. Kumamoto *et al.*, has measured a response of each muscle using Electromiogram (EMG) on the static condition in order to know the function of each muscle respectively. The experiment was excused in the following way[1][2]. An arm of a human subject fixed at a specific position as illustrated in Figure 1. Then, the subject generated a force from a wrist point toward six-directions while the wrist position was fixed during the action. Six-directions are positive and negative ones parallel to the line connecting the shoulder and the wrist, parallel to the line connecting the shoulder and the elbow, to the line connecting the elbow and the wrist respectively. These directions of the force were measured by strain gauges and the actuating stimulus to each muscle in the moment was measured by EMG.

Right part of Figure 1 shows the result of each muscle response when the subject generated the force from the wrist point toward certain direction from D1 to D6 on the tip of the arm. Vector to one point of hexagonal shape from the wrist shows the direction and the magnitude of the force which can generate by the subject. It has been cleared that the subject controls the magnitude of each muscle in the simple way as illustrated in Figure 1 respectively in order to output a force from a tip point toward a certain direction [1][2].

B. Calculation of kinematics model for the force control cooperated with bi-articular muscles

Relationship between joint torques which are controlled by corresponding to individual drive for each joint and bi-articular simultaneous drive and the characteristics of the output force on the tip point is mathematically discussed in this section. Characteristics of force at the tip point with the bi-articular muscle mechanism can be kinematically calculated.

Figure 2 shows calculation model of 2-link arm. When the original point is set to the shoulder point, the coordinate on the tip point $\boldsymbol{x}$ is described as follow;

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{bmatrix} \tag{1}$$

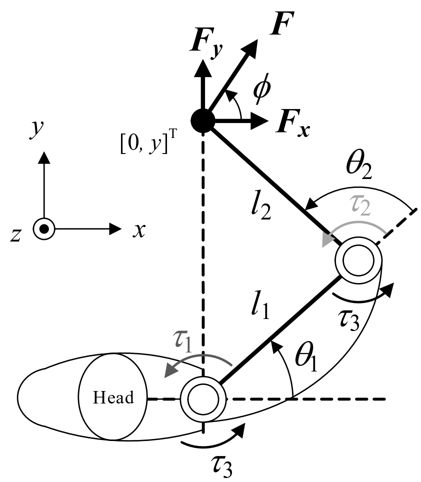

Figure 2. Fundamental arm model.

l_1, l_2 : Length of each link (m),
θ_1, θ_2 : Angle of each joint (rad)

Jacobian matrix $\boldsymbol{J}(\boldsymbol{\theta})$ on the tip point is calculated by the differential of angle as follow;

$$\begin{aligned} \boldsymbol{J}(\boldsymbol{\theta}) &= \begin{bmatrix} -\{l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)\} & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix} \\ &= \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \end{aligned} \tag{2}$$

There is a relationship between joint torques $\boldsymbol{\tau}$ and force of the tip point $\boldsymbol{F}$ from a principle of virtual work. Bi-articular muscles work both joints at the same time.

$$\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})^{\mathrm{T}} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{3}$$

In (3), joint torques $\boldsymbol{\tau}$ are described using three actuator torques. T_1 and T_2 mean the rotational torques by mono-articular muscles on shoulder point and elbow point respectively, and T_3 means the rotational torque by bi-articular muscles. When radii of rotations of each joint are assumed as same, (3) represents the simplified coordinates-transformation to the joint torques from these actuator torques[3]. The simultaneous drive of both joints by the bi-articular muscles is described by the factor at the 2th row and the 3th column in the transformation matrix.

The force on the tip point can be calculated from the inverse matrix of the Jacobian matrix. In the case that the length of each link is ($l_1 = l_2 = l$), the force of the tip point $\boldsymbol{F}$ is written as follows; Also, trigonometrical function is expressed as following values in this paper; where $\cos\theta_1 = \mathrm{C}_1, \sin(\theta_1+\theta_2) = \mathrm{S}_{12}$

$$\boldsymbol{F} = \frac{1}{l^2 \mathrm{S}_2} \begin{bmatrix} l\mathrm{C}_{12} & -l(\mathrm{C}_1+\mathrm{C}_{12}) \\ l\mathrm{S}_{12} & -l(\mathrm{S}_1+\mathrm{S}_{12}) \end{bmatrix} \boldsymbol{\tau} \tag{4}$$

TABLE I

PARAMETERS OF A 2-LINK ROBOT ARM.

θ_i	Angular position of joint i
m_i	Mass of link i
l_i	Length of link i
l_{gi}	Length between joint i and centre of gravity of link i
I_i	Moment of inertia through the centre of gravity of link i
τ_i	Rotational joint torques (τ_1, τ_2)
T_i	Each actuator torque (T_1, T_2, T_3)
g	Acceleration of gravity = 9.8 $(\mathrm{m/sec^2})$

C. Characteristic condition for bi-articular simultaneous drive

One specific condition for the bi-articular simultaneous drive is described in this section. If we consider that position of the tip point is at the y-axis in the Figure 2, relationship between each joint angle becomes as follow when length of each link is same;

$$2\theta_1 + \theta_2 = \pi, \;\; \mathrm{C}_1 + \mathrm{C}_{12} = 0 \;\text{ and }\; \mathrm{S}_1 = \mathrm{S}_{12}. \tag{5}$$

Therefore, force on the tip point $\boldsymbol{F}$ can be described as (6) by using actuator torques.

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{1}{l\mathrm{S}_2} \begin{bmatrix} \mathrm{C}_{12}(T_1 + T_3) \\ \mathrm{S}_{12}(T_1 - 2T_2 - T_3) \end{bmatrix} \tag{6}$$

Consequently, x-direction force becomes zero if $T_1 = -T_3$ and torque T_2 determines the magnitude of y-direction force directly. This result matches the previous research using EMG measurement and it can be considered that this situation is one of the simple control method in order to decide the force direction on tip point.

III. BI-ARTICULAR SIMULTANEOUS DRIVE FOR DYNAMIC MOTION CONTROL

A. Motion equations for the 2-link robot arm system

Characteristic of dynamic motion of a 2-link arm model is discussed in this section. Lagrange equation of motion has often used since conventional robotics control and this equation is also used in this paper.

Parameters for the calculation is shown as TABLE I. Length of each joint are same value as well as Section II. In addition, it is assumed that radii of rotations : $r_i = r$, mass of each link : $m_i = m$ and length between joint i and centre of gravity of link i : $l_{g1} = l_{g2} = \frac{1}{2}l$ respectively. Mass of load at the tip point is also considered as m_L. Length of load is defined as l_L and centre of gravity of load is defined as $l_{gL} = \frac{1}{2}l_L$. Radius of rotation is defined as same as each link. A cylindrical shape for each link and the load are assumed. Therefore, moment of inertia of each link and the load are described as follows;

$$I_1 = I_2 = m\Big(\frac{r^2}{4} + \frac{l^2}{12}\Big), \;\; I_L = m_L\Big(\frac{r^2}{4} + \frac{l_L^2}{12}\Big) \tag{7}$$

Length between joint2 and centre of gravity of link2 with load is calculated as follow;

$$l_{g2L} = l_{g2} + \frac{m_L}{m_2 + m_L}(l_{g2} + l_{gL}) \tag{8}$$

Therefore, moment of inertia of link2 with load can be described as follow;

$$\begin{aligned} I_{2L} &= I_2 + m_2\Big(\frac{m_L}{m_2 + m_L}(l_{g2} + l_{gL})\Big)^2 \\ &+ I_L + m_L\Big(\frac{m_2}{m_2 + m_L}(l_{g2} + l_{gL})\Big)^2 \end{aligned} \tag{9}$$

Equation of motion can be described as (10).

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}) \tag{10}$$

$\boldsymbol{M}(\boldsymbol{\theta})$ means an inertia matrix, $\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ means sum of a centrifugal and Coriolis forces, and also $\boldsymbol{g}(\boldsymbol{\theta})$ means gravity in the equation.

$$\boldsymbol{M}(\boldsymbol{\theta}) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$M_{11} = I_1 + m_1 l_{g1}^2 + I_{2L} + (m_2 + m_L)(l_1^2 + l_{g2L}^2 + 2l_1 l_{g2L}\mathrm{C}_2)$$

$$M_{12} = M_{21} = I_{2L} + (m_2 + m_L)(l_{g2L}^2 + l_1 l_{g2L}\mathrm{C}_2)$$

$$M_{22} = I_{2L} + (m_2 + m_L)l_{g2L}^2 \tag{11}$$

$$\boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} -(m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) \\ (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_1^2 \end{bmatrix} \tag{12}$$

$$\boldsymbol{g}(\boldsymbol{\theta}) = g\begin{bmatrix} m_1 l_{g1}\mathrm{C}_1 + (m_2 + m_L)(l_1\mathrm{C}_1 + l_{g2L}\mathrm{C}_{12}) \\ (m_2 + m_L)l_{g2L}\mathrm{C}_{12} \end{bmatrix} \tag{13}$$

From (10) - (13), inverse matrix of the inertia matrix can be calculated by hand. Hence, angular acceleration of shoulder and elbow joint can be also calculated[4].

B. Characteristic of an acceleration on the tip point

Variable numbers by actuator torques are considered in order to know the characteristics of the arm as well as Section II. In fact, angular accelerations of joints are calculated from Lagrange equation of motion. Therefore, characteristics of an acceleration of the tip of the arm can be also expressed by those values. Equation of the acceleration on the tip point in Figure 2 is shown as follow;

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \dot{\boldsymbol{J}}(\boldsymbol{\theta})\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{14}$$

Consequently, acceleration on the tip point can be calculated as (15), because determinant of the inertia matrix does not become zero.

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{M}(\boldsymbol{\theta})^{-1}\big(\boldsymbol{\tau} - \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) - \boldsymbol{g}(\boldsymbol{\theta})\big) + \dot{\boldsymbol{J}}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{15}$$

Finally, the two-dimensional tip acceleration is calculated in the following form;

$$\begin{aligned} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} &= \frac{1}{\Delta}\begin{bmatrix} J_{11}M_{22} - J_{12}M_{21} & -J_{11}M_{12} + J_{12}M_{11} \\ J_{21}M_{22} - J_{22}M_{21} & -J_{21}M_{12} + J_{22}M_{11} \end{bmatrix}\begin{bmatrix} \tau_{hg1} \\ \tau_{hg2} \end{bmatrix} \\ &- \begin{bmatrix} (l_1\mathrm{C}_1 + l_2\mathrm{C}_{12})\dot{\theta}_1^2 + l_2\mathrm{C}_{12}(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \\ (l_1\mathrm{S}_1 + l_2\mathrm{S}_{12})\dot{\theta}_1^2 + l_2\mathrm{S}_{12}(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \end{bmatrix} \end{aligned} \tag{16}$$

Here, determinant of the inertia matrix Δ, and a term of joint torque, a centrifugal force and Coriolis force, and acceleration of gravity are shown as following equations respectively;

$$\Delta = M_{11}M_{22} - M_{12}M_{21}$$

$$\begin{aligned}\tau_{hg1} &= T_1 + T_3 + (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) \\ &- g\{m_1 l_{g1}\mathrm{C}_1 + (m_2 + m_L)(l_1\mathrm{C}_1 + l_{g2L}\mathrm{C}_{12})\}\end{aligned}$$

$$\begin{aligned}\tau_{hg2} &= T_2 + T_3 - (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_1^2 \\ &- g(m_2 + m_L)l_{g2L}\mathrm{C}_{12} \qquad (17)\end{aligned}$$

C. Features of straight motion of the arm

In Section II, it has been discussed that x-direction force on the tip point could be easily controlled so far the tip point of the arm is kept on the y-axis. In the same way, characteristic of the acceleration of the tip while the tip moves straightly on the y-axis will be investigated in the following paragraphs. Acceleration of x-direction in (16) should be kept zero under this condition and velocity of x-direction on the tip point should be kept zero as well. Also, velocity on the tip point is calculated from time-derivative of the position of the tip point. While the tip of the arm is constrained and when the length of each link is identical as assumed in (5), the following equation of x-direction acceleration and x-direction velocity are derived;

$$\begin{aligned}\ddot{x} &= \frac{\Big((J_{11}M_{22} - J_{12}M_{21})\tau_{hg1} + (-J_{11}M_{12} + J_{12}M_{11})\tau_{hg2}\Big)}{\Delta} \\ &= 0 \qquad (18)\end{aligned}$$

$$\dot{x} = -l_1\dot{\theta}_1\mathrm{S}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2)\mathrm{S}_{12} = 0 \ \therefore\ 2\dot{\theta}_1 + \dot{\theta}_2 = 0. \qquad (19)$$

Therefore, the following relationship between τ_{hg1} and τ_{hg2} is necessary;

$$\frac{\tau_{hg2}}{\tau_{hg1}} = \frac{J_{11}M_{22} - J_{12}M_{21}}{J_{11}M_{12} - J_{12}M_{11}} \qquad (20)$$

In this situation, acceleration in y-direction is described from (16) as follow;

$$\begin{aligned}\ddot{y} &= \frac{1}{\Delta}(-J_{22}M_{21} + J_{22}M_{11}\frac{J_{11}M_{22} - J_{12}M_{21}}{J_{11}M_{12} - J_{12}M_{11}})\tau_{hg1} \\ &- 2l\mathrm{S}_{12}\dot{\theta}_1^2 \qquad (21)\end{aligned}$$

On the other hand, acceleration of y-direction is also described from the second order time-derivative of the position of the tip in (14);

$$\begin{aligned}\ddot{y} &= (l_1\mathrm{C}_1 + l_2\mathrm{C}_{12})\ddot{\theta}_1 + l_2\mathrm{C}_{12}\ddot{\theta}_2 \\ &- \{l_1\dot{\theta}_1\mathrm{S}_1 + l_2(\dot{\theta}_1 + \dot{\theta}_2)\mathrm{S}_{12}\}\dot{\theta}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2)\mathrm{S}_{12}\dot{\theta}_2 \\ &= l\mathrm{C}_{12}\ddot{\theta}_2 - 2l\mathrm{S}_{12}\dot{\theta}_1^2 \qquad (22)\end{aligned}$$

The valuable number τ_{hg1} can be described from (21) and (22) as following (23);

$$\tau_{hg1} = \frac{\Delta l\mathrm{C}_{12}\ddot{\theta}_2}{(-J_{22}M_{21} + J_{22}M_{11}\frac{J_{11}M_{22} - J_{12}M_{21}}{J_{11}M_{12} - J_{12}M_{11}})} \qquad (23)$$

From (17), (19) and (23), the sum of actuator torques $T_1 + T_3$ can be converted as follow;

$$\begin{aligned}T_1 + T_3 &= \frac{1}{2}\Big\{ - I_1 - m_1 l_{g1}^2 + I_2 + m_2(l_{g2}^2 - l_1^2) \\ &+ I_L + m_L(4l_{g2}^2 + 4l_{g2}l_{gL} + l_{gL}^2 - l_1^2)\Big\}\ddot{\theta}_2 \\ &+ g\{m_1 l_{g1}\mathrm{C}_1 + (m_2 + m_L)(l_1\mathrm{C}_1 + l_{g2L}\mathrm{C}_{12})\} \qquad (24)\end{aligned}$$

Consequently, it is obvious that the sum of actuator torques which are generated from shoulder muscle T_1 and bi-articular muscle T_3 do not have any nonlinear term, and value of angular acceleration of elbow $\ddot{\theta}_2$ and mass of load m_L becomes the parameters to design the sum of torques especially for the horizontal motion.

Subsequently, alternation of sum of actuator torques which is generated from elbow muscle T_2 and bi-articular muscle T_3 can be calculated from (17), (20) and (23);

$$\begin{aligned}T_2 + T_3 &= \frac{1}{2}\Big\{I_2 + m_2(l_{g2}^2 - l_1 l_{g2}\mathrm{C}_2) + I_L \\ &+ m_L(4l_{g2}^2 + 4l_{g2}l_{gL} + l_{gL}^2 - 2l_1 l_{g2}\mathrm{C}_2 - l_1 l_{gL}\mathrm{C}_2)\Big\}\ddot{\theta}_2 \\ &+ (m_2 + m_L)l_1 l_{g2L}\mathrm{S}_2\dot{\theta}_1^2 \\ &+ g(m_2 + m_L)l_{g2L}\mathrm{C}_{12} \qquad (25)\end{aligned}$$

Here, a load does not rotate with the link2 together, but exist as the role of an external disturbance as a special situation. In (25), moment of inertia of the load can be set to zero and centre of gravity of the load can bet set to zero as well. At the situation, the term of including the mass of load m_L are assumed as b and the other terms are also assumed as a in this paper. In addition, the mass of load on the tip point is considered as following equation;

$$m_L = (1 + k)m_3 \qquad (26)$$

Parameter m_3 is based on the assumption that one mass point on the tip of link2, and then the mass of load m_L is normalized by the hand mass m_3 using factor k. From the assumption, these two terms a and b are written as following equations;

$$a = \frac{1}{2}\{I_2 + m_2(l_g^2 - ll_g\mathrm{C}_2)\}\ddot{\theta}_2 + m_2 ll_g\mathrm{S}_2\dot{\theta}_1^2 + gm_2 l_g\mathrm{C}_{12} \qquad (27)$$

$$b = m_3\{\frac{1}{2}l^2(1 - \mathrm{C}_2)\}\ddot{\theta}_2 + l^2\mathrm{S}_2\dot{\theta}_1^2 + gl\mathrm{C}_{12}\} \qquad (28)$$

Furthermore, $a - b$ is calculated as following equation in case of $m_2 = 2m_3$; The condition of $m_2 = 2m_3$ is based on the following assumption;

- m_3 is supposed to mass of a hand. - Namely, muscles which are connected to shoulder and elbow moves an upper arm, a forearm and a hand.
- The centre of gravity of each link is at the middle point of its link respectively.
- Mass of a hand of a human is close to half value of human's forearm[6].

$$a - b = (\frac{1}{2}I_2 - \frac{1}{4}m_3 l^2)\ddot{\theta}_2 \qquad (29)$$

Therefore, the joint torque of the elbow point is simply expressed by parameter a which means characteristics of links, *i.e.*, moment of inertia of link and the value of m_3.

$$\begin{aligned} T_2 + T_3 &= a + (1+k)b \\ &= (2+k)a - (1+k)\Big(\frac{1}{2}I_2 - \frac{1}{4}m_3 l^2\Big)\ddot{\theta}_2 \end{aligned} \quad (30)$$

However, consideration of moment of inertia or rotation of the load is needed when the load moves simultaneously with link2.

IV. Characteristics of Joint Torques with Consideration of Equilibrant Force on the Tip Point Against an External Force

In general, d'Alembert's principle is used for the expansion of principle of a virtual work and it can be applied for dynamic calculation. First, d'Alembert's principle is simply explained.

Sum of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system, is zero.

$$\boldsymbol{F} - m\frac{d^2\boldsymbol{r}}{dt^2} = 0 \quad (31)$$

The total virtual work of the impressed forces plus the inertial forces vanishes for reversible displacements. This shows that the expansion of a principle of virtual work is possible to dynamic condition by the principle[5].

Hence, characteristics of straight motion of 2-link arm is descried finally as the following equation from (6), (24) and (25);

$$\begin{bmatrix} T_1 + T_3 \\ T_2 + T_3 \end{bmatrix} = \begin{bmatrix} \text{eq.}(24) \\ \text{eq.}(25) \end{bmatrix} + \begin{bmatrix} -2l\mathrm{S}_{12} & 0 \\ -l\mathrm{S}_{12} & l\mathrm{C}_{12} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad (32)$$

From (32), joint torques especially for the straight motion can be considered as following the three parts. Dynamic motion of y-direction in Figure 2 means horizontal motion of the arm. If we consider that the tip of the arm moves to z-direction straightly from the shoulder point in Figure 2, it means the vertical motion and we have to think about the gravity.

- On horizontal motion, the angular acceleration of elbow point $\ddot{\theta}_2$ and the mass and the size of the load on the tip point are the variable numbers in order to determine the joint torque of shoulder point. And $\ddot{\theta}_2$ is simply set from the sum of torques T_1 and T_3, because the characteristic of arm does not influence non-linearly in this case.
- Input torque at the elbow point τ_2 can be set easily from (30), when value of mass of load on the tip point is known, in other words, k is known before the motion. However, complicated control for non-linear terms due to joint angles or angular velocity are needed.
- Actuator torques which compensate the external force $\boldsymbol{F}$ during dynamic motion can be calculated using the relationship between each muscle force which produces joint torques $\boldsymbol{\tau}$, and the external force on statistic condition in Section II. For instance, a force sensor or a strain gauge are needed for external force applied to the tip point. The external force is compensated by using bi-articular simultaneous drive in this case.

The 3rd part means a principle of superposition directly, which can use the benefit of bi-articular simultaneous drive for the external force. For the motion control, control of each actuator torque T_1, T_2 and T_3, in other words, control of each muscle force is relevant to angular acceleration of joint and mass of load directly.

Moreover, it is considered in the 2nd part that actuator torques which are needed for the rotation of the elbow joint, are able to control by learning control. If it is possible to compensate those non-linear values from characteristic of inertia of the arm using learning control, motion control would be easier compare to conventional method. In addition, when we think of the motion of vertical direction, it can be considered that control of additional joint torques for the compensation of the acceleration of gravity would be also designed from learning control. The controller needs the information of joint angles at the starting of the learning control, but it would be able to compensate the non-linearity of inertia of dynamic motion or gravity for straight motion control. Because, the equation in order to realize the straight motion is not so complicated as shown in this paper.

V. Actuator Torques with Bi-articular Simultaneous Drive

In this section, bi-articular simultaneous drive in static condition is concretely described. The force on the tip point $\boldsymbol{F}$ has been calculated from (2), (3) and (6) in Section II, where joint angle information is always needed. In this paper, calculation using the information of the response of each muscle proposed and a simple actuator torque design is described.

First, the force on the tip point $\boldsymbol{F}$ is written by the magnitude F and direction ϕ as illustrated in Figure 2. In other words, F_x and F_y is described as $F_x = F\mathrm{C}_\phi, F_y = F\mathrm{S}_\phi$. Although a human has different maximal muscle forces in fact, the maximal actuator torques from three pair of muscles as assumed identical in this paper.

As an example, cases of $\frac{3}{2}\pi \le \phi < 2\pi - \theta_1$, $\frac{\pi}{2} \le \phi < \pi - \theta_1$ are mathematically considered. The clockwise rotational direction is defined as a positive, *i.e.*, the extensor muscles generate positive rotational torques and the flexor muscles generate negative torques respectively. In these cases, it can be desinged that amplitude of actuator torques T_1 and T_2 are desinged as same based on the EMG result. Consequently, the valuable numbers T_1 and T_3 are calculate as follows from (6) respectively when the magnitude of the force is F.

$$F\mathrm{C}_\phi = -\frac{1}{2l\mathrm{S}_1}(T_1 + T_3),\ F\mathrm{S}_\phi = \frac{1}{2l\mathrm{C}_1}(T_1 + 2T_1 - T_3) \quad (33)$$

$$\begin{aligned} T_1 &= \frac{lF}{2}(-\mathrm{S}_1\mathrm{C}_\phi + \mathrm{C}_1\mathrm{S}_\phi),\ T_2 = -T_1, \\ T_3 &= \frac{lF}{2}(3\mathrm{S}_1\mathrm{C}_\phi + \mathrm{C}_1\mathrm{S}_\phi) \end{aligned} \quad (34)$$

Then, the value of ϕ changes from $\frac{\pi}{2}$ to $\pi - \theta_1$, *i.e.*, direction of the force changes from D1 to D2 as illustrated in Figure 1.

In the same way, cases of $\pi + \theta_1 \leq \phi < \frac{3}{2}\pi$, $\theta_1 \leq \phi < \frac{\pi}{2}$ are also mathematically considered. Each actuator torques can be designed using force characteristics F and ϕ, and variable numbers of actuator torques T_1 and T_3.

$$FC_\phi = -\frac{1}{2lS_1}(T_1 + T_3),\ FS_\phi = \frac{1}{2lC_1}(T_1 - 2T_3 - T_3) \quad (35)$$

$$T_1 = \frac{lF}{2}(-3S_1C_\phi + C_1S_\phi),\ T_3 = -\frac{lF}{2}(S_1C_\phi + C_1S_\phi),$$
$$T_2 = T_3 \quad (36)$$

The cases of $2\pi - \theta_1 \leq \phi < 2\pi$ and $0 \leq \phi < \theta_1$, $\pi - \theta_1 \leq \phi < \pi + \theta_1$ are also able to calculated by the same consideration using EMG result. Finally, it has been considered that each actuator torque is ablee to designed by simple rule using EMG result.

Combined simultaneous control using two variable actuator torques which have same amplitude and one variable actuator torque depending on the value ϕ are actually described, when maximum value of each muscle force is assumed as same and its alternation is controlled using the result of EMG in Figure 1. Furthermore, if its motion controller knows the relationship between the amplitude of force F and angle of shoulder point θ_1 before the action, each actuator torque is controlled based on the information on ϕ.

Those calculations are valid only if the tip point is kept on the y-axis. Therefore, the joint angle of elbow θ_2 could be calculated from (5), and the relationship between the amplitude of the force F and the shoulder angle θ_1 was described as a simple equation. Whereas, information of shoulder and elbow angles are needed respectively, when the tip is not on y-axis, and the calculation of each actuator torque depending on the amplitude F and the direction of the force ϕ becomes more difficult than calculated case in this paper.

Compensation control of actuator torques based on the relationship between the magnitude of force F and angle of shoulder point θ_1 in case of discussed above might be possible, when the tip does not y on the y-axis. The characteristics of bi-articular simultaneous drive represents the simplified coordinates-transformation to the joint torques from the point of composition of the robot arm, and it would be used for easy control without real-time joint angle information.

VI. Conclusion

In this paper, differences between biological arm/legs and humanoid robots have been discussed from the point of its mechanisms and the control methods. Existence of bi-articular muscles and its simultaneous actuation to two joints are significant characteristics in biological objects. The mathematical meaning of the bi-articular simultaneous drive was described in Section II.

When the analysis of the motion is specifically limited to the case that link length and radii of joints are same values respectively, and the tip of the 2-link arm moves straightly on y-axis in Figure 2, the characteristics of actuator torques and the force on the tip point are substantially simplified. On horizontal motion as illustrated as y-axis direction in Figure 2, it has been mathematically confirmed that the sum of two-actuator torques on shoulder point does not have non-linear terms itself, and the torques can be simply designed based on the angular acceleration of elbow point $\ddot{\theta}_2$ and the mass and the size of a load on the tip point.

In addition, each actuator torque with bi-articular simultaneous drive can be determined following the measured results of a human arm using Electromiogram in Figure 1. A control using the relationship between the force on the tip point and actuator torques has been proposed. Under the assumption that maximal value of each muscle is the same, two valiable actuator torques which have same amplitude and one variable torque depending on the angle of the external force ϕ can be applied based on the EMG-measurement results. In aditiion, the amount can be directly determined from the amplitude of the force F and joint angles.

As a future work, compensation of the error from the proposed actuator control on static and dynamic motions will be considered. Actuator control to compensate the external force using bi-articular simultaneous drive is realized mathematically when the tip of the arm is kept on y-axis, and learning control in order to get back the position in the case that the position is changed by the external force. Application to the flexibility of muscle model can be also taken into consideration for it. Furthermore, designing of learning control to make a trajectory with consideration of inertia of arm will be also considered collaboration with the straight motion characteristic in this paper.

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