Efficient Algorithm for Evaluating and Optimizing Train Reschedules by Taking Advantage of Flexibility of Quadruple Track

Kazuhiro Hara[1], Kazumasa Kumazawa[1], Takafumi Koseki[2]

Abstract Rescheduling train operation is often needed when train operation is disordered by physical injuries, vehicle malfunction, signal troubles, and so on. Main tasks for it are presently taken by train dispatchers with their experience and intuitions. It is hard to reschedule train operation in short time, considering many factors concerning train operation without any established algorithm for creating a new appropriate plan or evaluating the quality of the proposed plan.

The authors' group has developed a system for supporting the task of the train rescheduling by proposing and evaluating a new plan of train operation.

In this paper, the authors extend the target model line to quadruple track where the task is substantially more complicated and smart supporting system is especially requested. The authors have implemented several methods for train rescheduling by taking advantage of flexibility of quadruple track and examined these effects under various conditions. The authors furthermore propose an efficient algorithms for evaluating and optimizing train rescheduling plans to make them in a practical time.

Keywords : Train rescheduling, Support system, Evaluation, Quadruple track

1. INTRODUCTION

Trains are operated according to the well-designed diagram. Therefore, once small disruption is occurred by physical injuries, vehicle malfunction, signal troubles, and so on, great confusion might be caused because the disruption affects various parts of diagram. Changing diagram appropriately is needed to prevent spreading the influence when the train operation is disordered. This work is called train rescheduling.

Main tasks for it are presently taken by train dispatchers. It is very difficult, even for experienced train dis- patchers, since there are many factors to be considered such as location of trains and rolling stocks, layout of stations, location of drivers, passengers' demand, and so on. Furthermore, rescheduling must be done in short time under uncertain information. Train dispatchers deal with the task with their experiences and intuitions since there is no established algorithm for creating new appro- priate plan or evaluating a new plan of train operation. So a system supporting train dispatchers' task is requested (1).

The authors' group proposed a method to rate train operation plans quantitatively from passengers' point of view. The authors furthermore developed a system for supporting the task of the train rescheduling by proposing and evaluating a new plan of train operation (2).

In this paper, the authors extend the target model line to quadruple track, especially to direction quadruple track illustrated in figure 1, where the task is substantially more complicated and smart supporting system is especially requested. Train operation can be continued even if one track becomes interrupted in the quadruple track section because there are two same directional tracks. So a lot of coping strategies exist in quadruple track section compared with single or double track section.

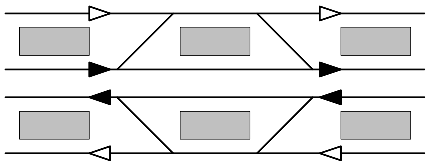

Fig. 1 Direction quadruple track whose operational freedom as well as complexiety are large

The authors have implemented several methods for train rescheduling by taking advantage of flexibility of quadruple track and examine these effects under various conditions. The authors furthermore propose an efficient algorithm for evaluating and optimizing train rescheduling plans to make them in a practical time. And the authors show case studies to verify the performance of the proposed methods.

2. COMPOSITION OF TRAIN RESCHEDULING SYSTEM

A rescheduled plan is made by the supporting system based on the information of delay and various restrictions.

The system consists of two major modules, one is creating plan and the other is evaluating the plan. Change of diagram and determination of arrival or departure time are executed in the creation part. Estimation of passengers' behavior and calculation of evaluation indices are carried out in the evaluation part. A good plan is rescheduled by repeating process of adoption judgment of a new modification based on the evaluation indices. And the rescheduled plan is presented to the train dispatchers.

Figure 2 shows a composition of the computer-aided train re-scheduling system.

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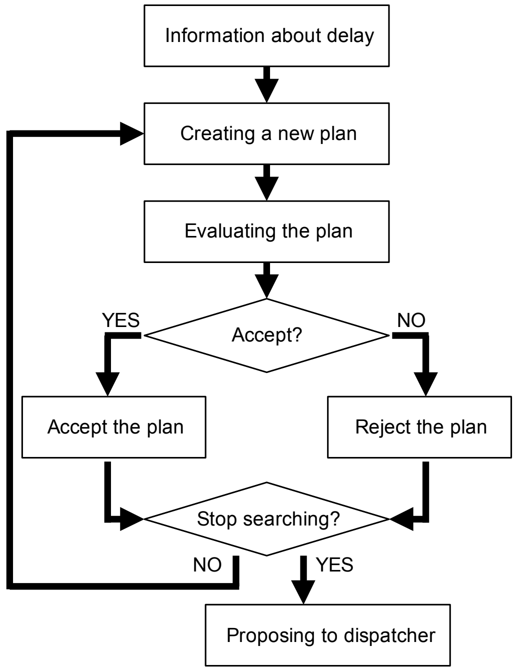


Fig. 2 Algorithm of the computer-aided rescheduling system

3. CREATION OF TRAIN OPERATION PLAN

Creating train operation plan on the system consists of application part of train rescheduling methods and decision part of arrival or departure time by simulation of train operation.

3.1 Application of train rescheduling methods When train operation is disordered, train rescheduling methods are applied to prevent spreading of delay. It is important to select appropriate methods in order to attain desired effect.

Figure 3 shows a change of overtaking station as an example of train rescheduling methods. Figure 3 (a) shows an initial schedule, an express train overtakes a local train at station A. Figure 3 (b) shows the case the express train is delayed. The local train waits for the arrival of the express train for a long time at station A, so it is affected of the delay considerably. Figure 3 (c) shows the case that change of overtaking station is applied to (b). The influence of delay affecting to the local train is limited by the change of overtaking station to station B. Change of train order or track is executed only in this part, arrival or departure time is not determined.

3.2 Simulation of train operation There are various constraints for train operation as follows.

(1) Regular departure or arrival time A train must not run earlier than a regular schedule.

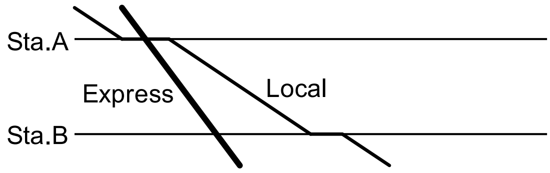
(2) Regular running time To run between stations, a train requires longer time than regularly defined by the type of rolling stock, number of cars, stop or pass of stations, and so on.

(3) Regular stopping time A train stops at the station longer than the defined time.

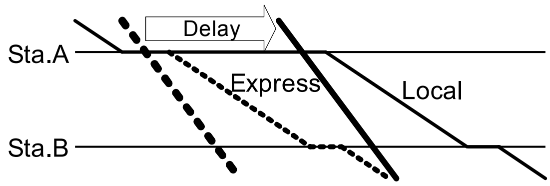
(4) Order of trains A train runs and stops in the order corresponding to schedule.

(5) Blockage Trains more than defined number can not run at the same time between stations.

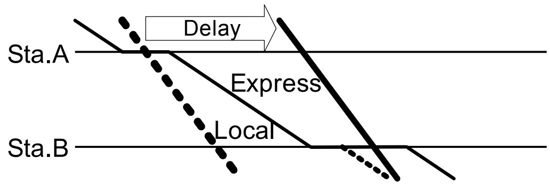
(6) Confliction of routes A certain interval is required between trains whose routes conflict at a station.



(a) initial schedule



(b) Delay of express



(c) Change of overtaking station

Fig. 3 An example of typical train rescheduling methodology

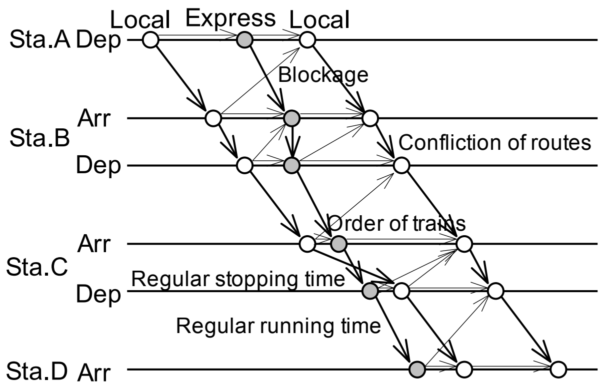


Fig. 4 An example of a graph representing train operation

The authors use graph structure and the longest path search to decide arrival or departure time filling all constraints. Nodes of a graph express departure or arrival of trains at each station. Links represent running and stopping of train, and constraints mentioned above. The weight of the links are requisite interval for filling constraints. By finding the longest path to each node from the node expressing origin of time, it is possible to obtain departure or arrival time of each train of each station as the length of the path (3). The authors use PERT(Program Evaluation and Review Technique) for the longest path search. It is necessary to change the information on nodes or links appropriately to express the application of train rescheduling techniques. Figure 4 shows an example of a graph representing train operation.

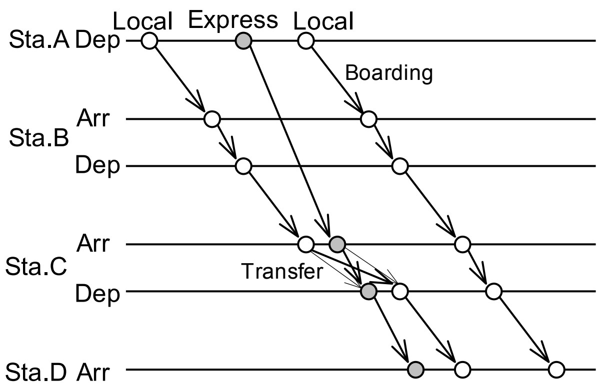


Fig. 5 An example of a graph representing passengers' flow

4. EVALUATION OF TRAIN OPERATION PLAN

The total time of all trains' delay has been used for the evaluation of the train rescheduling plan in practical use. Because the delay is vanished when all delayed trains are suspension of service, it is evaluated as a good train rescheduling plan in this criterion. However, the passengers overflow on the platform actually, it is clear that this rescheduling plan is not good.

In order to avoid such stupid results, the authors' group has proposed a mehod to rate train operation plans quantitatively from passengers' point of view. This section describes how to estimate passengers' flow and evalutaion indices.

4.1 Simulation of passengers' flow The authors assume following model of passengers' behavior. Each passenger appears at his/her starting station uniformly, and he/she selects lines and trains so as to minimize his/her loss caused by traveling time and transfer. This model represents passengers who appear at stations without checking train timetable and know information about lines. Such passengers are often seen in urban railways.

The authors use graph structure and the shortest path search to estimate lines and trains each passenger selects based on the models of passengers' behavior. Nodes of a graph express departure or arrival of trains at each stop. Links describe potential passengers' flow between departure and arrival. The weight of the links representing boarding are running time between stations or stopping time at a station. The weight of the links representing transfer are the summation of the time required for the transfer and time converted loss of it.

The path on which total loss is the least can be calculated by applying Dijkstra algorithm. It is possible to estimate the number of the passengers boarding on each train by allocating passengers to the links by using OD(Origin-Destination) table.

Figure 5 shows an example of a graph representing passengers' flow under the same train operation as shown in figure 4.

4.2 Evaluation of train operation from passengers' point of view The authors defined a criteria for passengers' loss considering the following three evaluation indices, traveling time, burden of transfer, and congestion. These indices are calculated by using the result of simulation of passengers' flow. For the purpose of adding up these three types of loss calculated by different measures, latter two types of losses are converted into equivalent time.

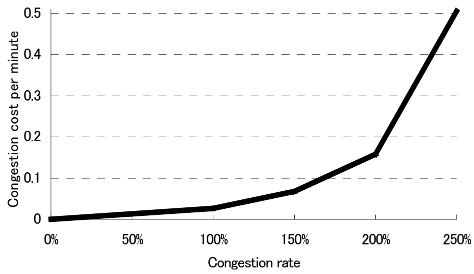


Fig. 6 Nonlinear relationship between congestion rate and congestion cost per minute

(1) Travaling time The most significant factor for passengers to evaluate train operation is amount of time required from the starting station to the destination.

Traveling time is defined as follows.

$$L_1 = \sum_{i=1}^{N} t_i \quad (1)$$

where N is the number of passengers, t_i is passenger i's traveling time.

(2) Burden of transfer Transfer not only takes time but also imposes burden such as going upstairs. These burdens are calculated as passengers' loss in addition to the real time for transfer.

Burden of transfer is defined as follows.

$$L_2 = \sum_{i=1}^{N} \sum_{j=1}^{M_i} r_{ij} \quad (2)$$

where M_i is the number of transfers passenger i needs, r_{ij} is time equivalent to the burden of passenger i's j-th transfer.

(3) Congestion Congestion of trains is also evaluated as loss because passengers in a congested train feel discomfort.

Congestion loss is defined as follows.

$$L_3 = \sum_{k=1}^{n-1} \sum_{s=1}^{S_k} f_c\left(\frac{q_{ks}}{c_{ks}}\right) q_{ks} t_{ks} \quad (3)$$

where n is the number of stations, S_k is the number of trains which arrive at station k. q_{ks}, t_{ks}, and c_{ks} are the number of passengers in the train, the time required, and the capacity of the train between k-th and k+1-th stations respectively. f_c is a nonlinear function to convert passengers' discomfort in the congested train into equivalent time (4), as shown in figure 6.

The evaluation criteria of a train operation is the sum of above-mentioned three types of loss.

$$L = L_1 + L_2 + L_3 \quad (4)$$

5. EFFICIENT ALGORITHM BY TAKING ADVANTAGE OF QUADRUPLE TRACK

It is important to make train rescheduling plan quickly though it is a complex combinatorial optimization problem. Since the number of intended trains increase in a quadruple track section, the problems become more complicated and it takes more computational time.

Then, the authors propose a method of making rescheduling plan without multiple search on a minor change of schedule. This provides room for considering the combination of various rescheduling methods.

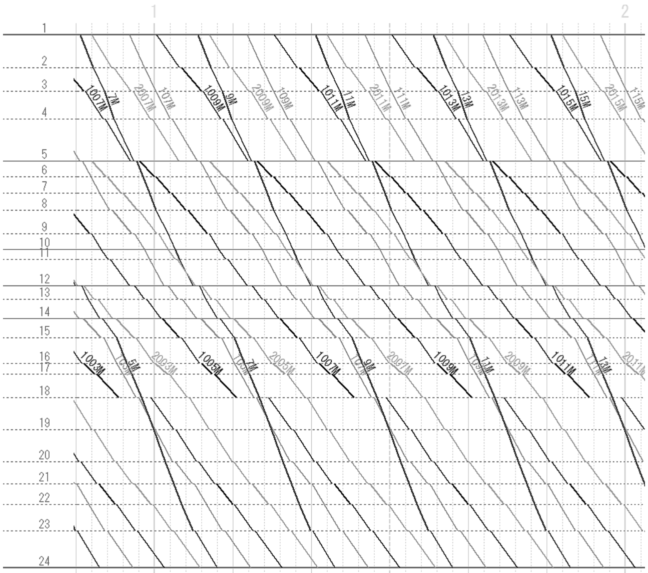


Fig. 7 Regular schedule of model line

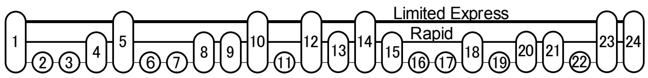
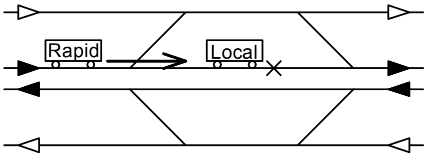
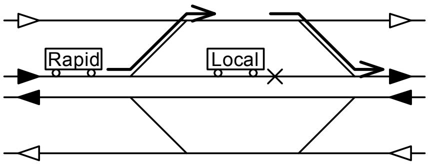


Fig. 8 Stopping pattern of trains



(a) Before change of running tracks



(b) After change of running tracks

Fig. 9 Schematic diagram of change of running tracks

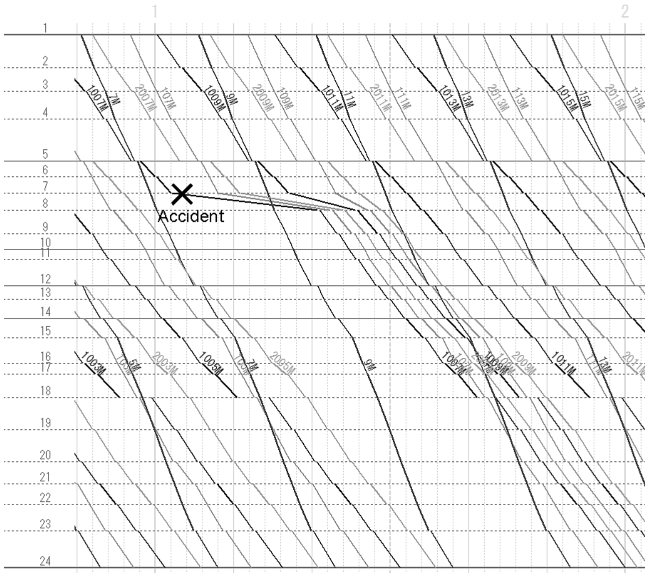


Fig. 10 After an accident

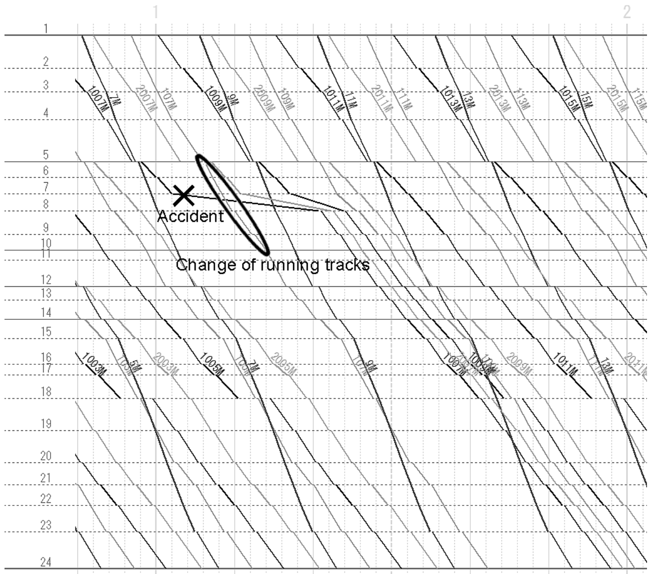


Fig. 11 A rescheduled plan improved by change of running tracks

5.1 Feature of model line Figure 7 shows regular schedule of model line and figure 8 shows stop pattern on the line. The features of this line are as follows.

- All sections are quadruple track.
- Limited express trains run faster track, rapid and local trains run slower track.
- 1 limted express train, 1 rapid train and 2 local trains run in 15 minute periods.
- A limited express train and a local train make connection at station 5 and 12, and a rapid train and a local train make connection at station 5 and 18.

6. CHANGE OF RUNNING TRACKS

6.1 Change of running tracks Change of running tracks is a method for train rescheduling to continue train operation by changing running track to another track. It is relatively easy in quadruple track because there are two same directional tracks. It can be said that it is a method taking advantage of quadruple track most.

Figure 9 shows a schematic diagram of the change of running tracks.

6.2 Effect of change of running tracks Figure 10 shows a situation that 1007M, local train, delays for 1000 seconds between station 7 and 8. The influence reaches to following rapid and local trains because the local train using the slower track has stopped. As for 9M, limited express, it can run as usual because it uses the faster track.

Figure 11 shows a diagram in which change of running tracks is applied to 107M, rapid train. It runs on the faster track between station 5 and 10. The loss time of the rapid train's passengers can limit by change of running tracks. Figure 12 shows passengers' loss. It shows that the schedule is improved by change of the running tracks because passengers' loss is decreased.

The sections to which the change of running tracks can be applied are limited because crossovers between the faster and the slower tracks are needed.

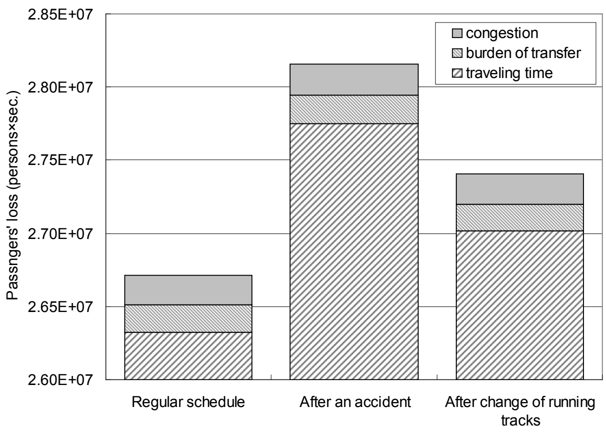


Fig. 12 Comparison of passengers' loss

7. EXTRA STOP

7.1 Effect of extra stop The interval of train stopping at a station can be long as results of an accident and a train rescheduling. Therefore, the loss time of passengers going to the station or getting on from the station is large.

A method for giving relief to the passengers by additional train stops is called extra stop. It is not executable at all station because there are stations without platform in the faster track.

7.2 Efficient method of searching extra stop station It is a simple mean to calculate each evaluation index of all candidate stations for an extra stop, as a method of searching the best station and train of extra stop. However, the computing time increase by this means because simulations of train operation and passengers' flow are needed a number of times.

The authors, therefore, propose a method to evaluate the operation efficiently. The loss time is calculated by using the number of passengers obtained by simulation of passengers' flow in this method. Figure 13 and formulae defined below represent effects of the extra stop.

(1) Passengers who transfer trains at station A, and arrive at station B by a following train The passengers' traveling time shorten by t_2, and burden of transfer r_α vanishes. Assuming that number of corrresponding passengers are P, loss time is decreased as follows.

$$T_1 = \sum_{i=1}^{B-1} P_i (t_{2i} + r_{\alpha i}) \quad \cdots\cdots (5)$$

(2) Passengers who get on a train from station B The passengers become possible to get on a train which departs station B earlier than former schedule by the extra stop. Assuming that passengers going to station j appear q_j person per second, loss time is decreased as follows.

$$T_2 = \sum_{j=B+1}^{N} q_j t_{1j} (t_{3j} + r_{\beta j}) \quad \cdots\cdots (6)$$

where t_{1j} is interval between the express and preceding reasonable train, t_{3j} is arrival time difference to station j, $r_{\beta j}$ is difference of burden of transfer.

(3) Passengers who have got on the express and pass the station B The passengers' traveling time increase t_4 by acceleration, deceleration and stop. Assuming that number of corresponding passengers are R, loss time is increased as follows.

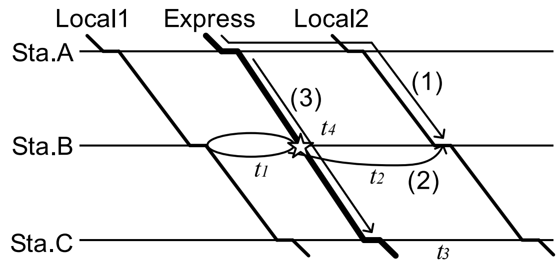


Fig. 13 Effect of an extra stop

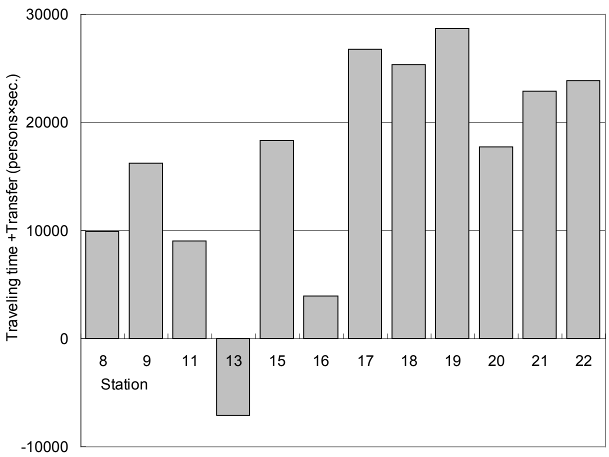


Fig. 14 Passengers' loss by changing extra stop station

$$T_3 = \sum_{i=B+1}^{N} R_i t_4 \quad \cdots\cdots (7)$$

Thus difference of loss time by extra stop to station B is calculated as follows.

$$\Delta T = -T_1 - T_2 + T_3 \quad \cdots\cdots (8)$$

7.3 Effect of proposed method The authors have applied the proposed method to the diagram shown in figure 11 in order to verify the effect.

Local trains do not arrive for a long time after the limited express 9M. Therefore, it is thought that the loss time for waiting for a following train, mentioned in 7.2 (1), is especially large. The increase and decrease of passengers' loss when 9M stops temporarily at its nonstop station is calculated, and the train is determined to stop at the station where decreased loss is maximum. The passengers' total loss has been reduced by this way.

Figure 14 shows variation of passengers' loss by extra stop on the example shown in figure 11. The loss time is increased in the most cases. This is because the limited express delays by the influence of the extra stop explained in 7.2 (3). However, the total passengers' loss can be decreased by the extra stop to station 13. This is because there are a lot of passengers who go to station 13, and waiting time for transfer at station 12 is decreased.

The computational time has been reduced to approximately 1/10 by proposed method compared to the conventional numerical approach. This contributes to the entire speed-up of making rescheduled plan.

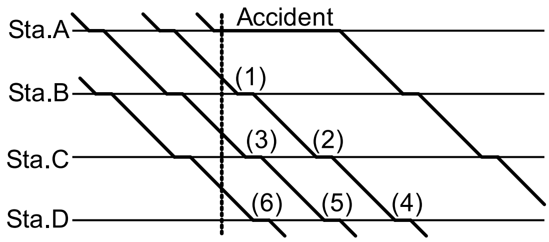


Fig. 15 Applying point of interval adjustment

8. INTERVAL ADJUSTMENT

8.1 Effect of interval adjustment Passengers usually take first-departing train without checking timetable in advance in the urban area. A number of passengers of each train is decided by interval from preceding train in such lines. When a train delays, the number of passengers waiting for the train increases. Therefore, further delay is caused by additional passengers' getting on and off.

To prevent this situation, passengers are dispersed by stopping the precedent train intentionally longer to keep interval equality. Early recovery of diagram is intended by doing so. This method is called interval adjustment.

It is important how many seconds to increase the stopping time of the train for the interval adjustment. The most reliable method of finding the best executing time is to vary the executing time at all target point. However, the computing time becomes huge by this method. So the authors propose a method searching from the most significant point in sequence, as human intuitively thinks. The search is executed in sequence from (1) to (6) in an example as shown in figure 15. A fairly good result was obtained by proposed method compared with full search, and computing time was able to be shortened drastically.

8.2 Efficient searching method for amount of interval adjustment using analytical solution At first, the authors used one-dimensional search using golden section ratio known as an efficient algorithm to calculate the best execution time. However, multiple simulation of train operation and passengers' flow are needed on a process of one-dimensional search.

There is a case where the best execution time can be calculated analytically by using number of passengers and train intervals, without multiple change of diagram. The authors aim to shorten computational time more by above-mentioned method. Figure 16 and following formulae show how to calculate the best interval adjustment time of train b at station A.

(1) Passengers who get on the train and pass the station A

The passengers' traveling time increase by interval adjustment at station A. Assuming that interval adjustment is done for x seconds, loss time is increased as follows.

$$T_1 = P_k(x - \tau_{ik}) \quad \cdots (9)$$

where P_k is number of passengers going to station k, τ_{ik} is margin time from station A to k.

(2) Passengers who transfer to the train on the way Loss time is increased as follows in this type of passengers.

$$T_2 = Q_{jk}(x - \tau_{jk}) \quad \cdots (10)$$

where Q_{jk} is number of passengers going from station j to k, τ_{jk} is margin time from station j to k.

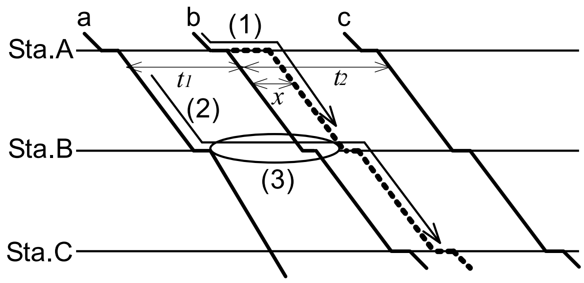


Fig. 16 Effect of an interval adjustment

(3) Passengers who enters from the station interval adjustment has been applied and the following stations, aim further station The authors use a model that each passenger appears at his/her starting station uniformly. The summation of passengers' waiting time before interval adjustment is shown in formulae 11 and 12, and after it is shown in formulae 13 and 14. The loss time changed by interval adjustment is, therefore, shown in formula 15.

$$T_{31} = \frac{1}{2} p_{jk} t_{1j}^{\;2} \quad \cdots (11)$$

$$T_{32} = \frac{1}{2} p_{jk} t_{2j}^{\;2} \quad \cdots (12)$$

$$T_{33} = \frac{1}{2} p_{jk} \left\{ t_{1j} + \left(x - \tau_{ij}\right) \right\}^2 \quad \cdots (13)$$

$$T_{34} = \frac{1}{2} p_{jk} \left\{ t_{2j} - \left(x - \tau_{ij}\right) \right\}^2 \quad \cdots (14)$$

$$\begin{aligned} T_3 &= \left(T_{33} + T_{34}\right) - \left(T_{31} + T_{32}\right) \\ &= p_{jk} \left\{ \left(x - \tau_{ij}\right)^2 - \left(t_{2j} - t_{1j}\right)\left(x - \tau_{ij}\right) \right\} \end{aligned} \quad \cdots (15)$$

where p_{jk} is number of passengers going from station j to k per second.

There are passengers whose boarding train is changed from c to b by the interval adjustment. Their loss time difference is calculated in formula 16.

$$T_4 = p_{jk}\left(x - \tau_{ij}\right)\left\{ -t_{\beta jk} + \left(t_{\alpha jk} - \tau_{jk}\right) \right\} \quad \cdots (16)$$

The summation of passengers' loss changed by the interval adjustment is calculated in formula 17.

$$\begin{aligned} \Delta T &= \sum (T_1 + T_2 + T_3 + T_4) \\ &= \sum p_{jk} x^2 \\ &\quad - \left[\sum p_{jk} \left\{ \left(t_{2j} - t_{1j}\right) + \left(t_{\beta jk} - t_{\alpha jk} + \tau_{jk}\right) + 2\tau_{ik} \right\} - \sum \left(P_k + Q_{jk}\right) \right] x \\ &\quad + \sum p_{jk} \tau_{ij}^{\;2} + \sum p_{jk} \left(t_{2j} - t_{1j}\right) \tau_{jk}^{\;2} - \sum P_k \tau_{ik} - \sum Q_k \tau_{jk} \\ &\quad + \sum p_{jk} \tau_{ij} \left\{ t_{\beta jk} - t_{\alpha jk} + \tau_{jk} \right\} \end{aligned}$$

$$\cdots (17)$$

Since it is a second-order polynominal of x, the optimal x is obtained from the following equation.

$$x = \frac{\sum \left\{ p_{jk}\left(t_{2j} - t_{1j}\right) + \left(t_{\beta jk} - t_{\alpha jk} + \tau_{jk}\right) - \sum \left(P_k + Q_{jk}\right) \right\}}{2 \sum p_{jk}}$$

$$\cdots (18)$$

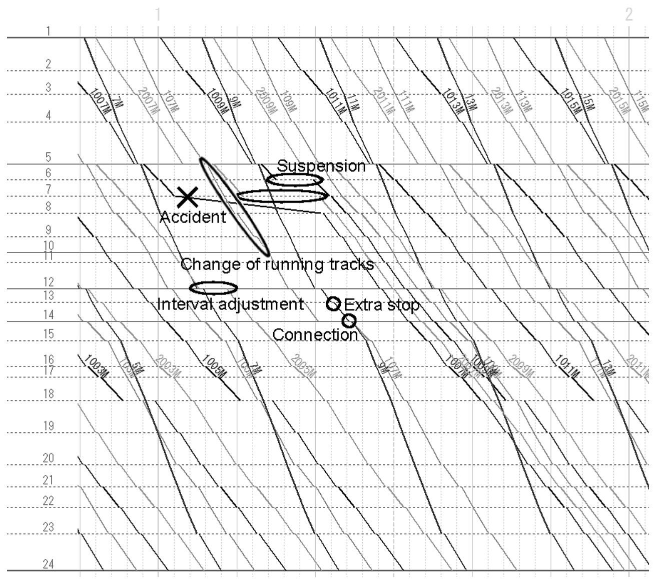


Fig. 17 A rescheduled result

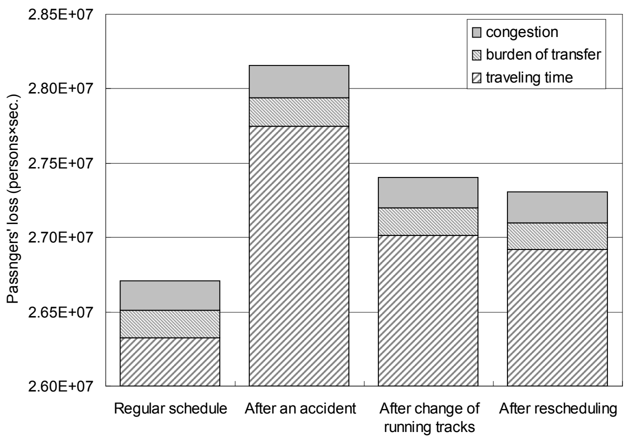


Fig. 18 Comparison of passengers' loss

The computational time has been reduced to further more approximately 1/10 by this analytical method.

Figure 17 shows a rescheduled plan made by the system and figure 18 shows passengers' loss of each schedule. The schedule with less passengers' loss is generated by combination of the various train rescheduling methods.

9. CONCLUSION

This paper describes an idea of evaluation index from passengers' point of view based on calculation of passengers' flow for supporting the task of the train rescheduling. It has also described an efficient algorithm for implementing the idea as a computer program concretely.

The authors have proposed a method of making rescheduling plan without multiple search for minor changes of schedule by straight forward analytical optimizing approaches for creating and evaluating the plan efficiently. And the authors verify the performance of proposed methods. The computational time has been shortened drastically by proposed method. This provides room for a large real-time rescheduling problem by combining various rescheduling methods and possibility of making good train rescheduling plan by avoiding local optimum.

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