Nonlinear Controller Design of Three Degree-of-Freedom Hybrid Magnetic Levitation Control Based on Fuzzy Model for a Contactless Servo-Actuator

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Abstract — **This paper presents a nonlinear servo-control design based on Fuzzy model for safer and comfortable levitation of an electromagnetic suspension stage consisting of triple arrangement of hybrid electromagnets, since conventional linear control design can give satisfactory results only around a specified linearization point for tiny displacements. The authors have proposed compensation principle for a proper control action according to change of operational points through fuzzy inference algorithm. Fuzzy reasoning acts as a gain scheduler among approximate local linear controllers derived from various gap operating points. The authors integrate a zero-order disturbance observer for estimating gap velocity and winding currents, as well as for improving performance of the system under external load-force changes. The effectiveness of the proposed control methodology is verified through several experiments.**

Keywords— **fuzzy model-based control, hybrid electromagnet, servo control, magnetic levitation, magnetic suspension**

I. Introduction

Electromagnetic suspension mechanism, in which attractive forces of electromagnets are used as suspension forces, is commonly used in passenger transport vehicles[1], due to several advantages, such as no friction, no abrasion, low noise and small vibration, *etc*.
Furthermore, recent industrial demands request relatively large change of gap length for contactless electromagnetic manipulation. Electromagnets are equipped in the system to provide redundancy and multiple degree-of-freedom manipulation capability.

Designing a controller satisfying predefined performance index is rather challenging task owing to apparent nonlinearity and instability of a hybrid electromagnet. In addition, when multiple electromagnets are used for suspending a body, the control design becomes more complicated due to interactions among hybrid electromagnets. A conventional approach is to design active controllers based on traditional linear control theory through local approximate linearization for simplicity. It has been assumed that the designed active controllers provide required performance criteria around a specified operating point for tiny displacement in such methods. The approach possibly threatens its stability when the operating point is far from the initially assumed operating point.

It is known that Takagi-Sugeno-Kang fuzzy model-based control overcomes the the nonlinear problem by preserving the easiness of linear controller designs through employing seamless transition among local linear controllers according to wide change of operating points[2].

Application of the fuzzy-model based control will be explained as a control design process, preserving simplicity of linear control and systematic extension of control design in this paper, in order to achieve robust stabilizing control for magnetic levitation of a stage suspended by three electromagnets. The effectiveness of the proposed control combined with a disturbance observer is demonstrated in an experimental study.

II. Control Plant: An Electromagnetically Suspended Stage

The electromagnetically suspended stage in this paper has three hybrid U-shaped electromagnets arranged on a stiff base in symmetrical manner with 120 degrees geometrical separation. Fig. 1 illustrates the suspended part of the system. Both redundancy and and multiple degree-of-freedom motion control capability including inclination angle axes, α and β, have been obtained. The attractive force produced between a U-shaped hybrid electromagnet and ferromagnetic table on ground can be expressed by assuming that;

(1) magnetic resistance of the iron core,
(2) flux leakage and fringing, and
(3) saturation, hysteresis and eddy current

are negligible [1]:

$$F_e = k\left(\frac{i+I_m}{x+L_m/\mu_{rm}}\right)^2 \quad \text{...(1)}$$

where k is the force coefficient correcting magnetic parameters, i is the coil current, I_m is the excitation current equivalent to permanent magnetomotive force, x is gap length including the thickness of plastic sheet cover for magnet's protection, L_m is the length of electromagnet and μ_{rm} is the relative permeability of the material of the permanent magnet. Local motion dynamics of a single hybrid electromagnet is represented by assuming the one-third of the total suspension mass is levitated associated hybrid electromagnet as [3],

$$m\ddot{x}_{A|B|C} = -k\left(\frac{i_{A|B|C}+I_m}{x_{A|B|C}+L_m/\mu_{rm}}\right)^2 + mg + \Gamma_{A|B|C} \quad .....(2)$$

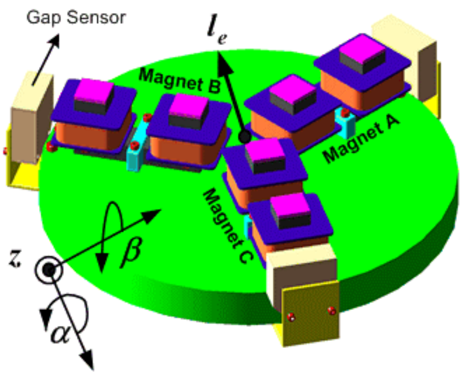


Fig. 1 Levitated body.

Notice that each hybrid electromagnet is identical to the others in physical quantities. On the other hand, electrical dynamics of the system is described by;

$$V_{A|B|C} = R i_{A|B|C} \sqrt{kS\mu_0}\frac{d}{dt}\left(\frac{i_{A|B|C}+I_m}{x_{A|B|C}+L_m/\mu_{rm}}\right) \quad ....(3)$$

where, $A|B|C$ -notation means corresponding hybrid electromagnet, m is one-third of the total levitation mass, g is gravitational acceleration, Γ is external load force treated as a disturbance[4], R is coil resistance, S is area of pole face, μ_0 is permeability of vacuum, and V and i correspond to applied voltage and current of the magnet coil, respectively. Approximate linearization is applied to investigate the behavior of the system and design a controller by using linear control theory.

$$K_a = \frac{-\partial F_e}{\partial x}\Big|_{x=x_0,\, i=i_0} \quad ....(4)$$

$$K_b = \frac{-\partial F_e}{\partial i}\Big|_{x=x_0,\, i=i_0} \quad ....(5)$$

$$F_e = F_e(x_0, i_0) - K_a\Delta x + K_b\Delta i \quad ....(6)$$

Consequently, linearized local motion dynamics of a hybrid electromagnet is derived as:

$$\frac{d}{dt}\begin{bmatrix}\Delta x\\ \Delta\dot{x}\\ \Delta i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{K_a}{m} & 0 & \frac{-k_b}{m}\\ 0 & \frac{K_a}{K_b} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta\dot{x}\\ \Delta i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ i\frac{1}{L}\end{bmatrix}\Delta V + \begin{bmatrix}0\\ \frac{1}{m}\\ 0\end{bmatrix}\Gamma \quad (7)$$

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} + B_d\boldsymbol{\Gamma} \quad ....(8)$$

$$y = [1\ \ 0\ \ 0]\begin{bmatrix}\Delta x\\ \Delta\dot{x}\\ \Delta i\end{bmatrix} = C\boldsymbol{x} \quad ....(9)$$

Moreover, to allow representation of the system dynamics in absolute axes, following representation matrices are defined for displacements and currents, respectively;

$$\begin{bmatrix}\Delta z\\ \Delta\alpha\\ \Delta\beta\end{bmatrix} = \begin{bmatrix}-\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3}\\ -\frac{2}{3l_e} & \frac{1}{3l_e} & \frac{1}{3_e}\\ 0 & -\frac{1}{\sqrt{3}l_e} & \frac{1}{\sqrt{3}l_e}\end{bmatrix}\begin{bmatrix}\Delta x_A\\ \Delta x_B\\ \Delta x_C\end{bmatrix} = \boldsymbol{T}\begin{bmatrix}\Delta x_A\\ \Delta x_B\\ \Delta x_C\end{bmatrix}, \quad (10)$$

$$\begin{bmatrix}\Delta i_z\\ \Delta i_\alpha\\ \Delta i_\beta\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}\Delta i_A\\ \Delta i_B\\ \Delta i_C\end{bmatrix} = \boldsymbol{H}\begin{bmatrix}\Delta i_A\\ \Delta i_B\\ \Delta i_C\end{bmatrix}. \quad (11)$$

Parameter set used in the study is summarized in Table 1 as follows.

Table 1 parameter set in the study.

Symbol	**Description**	**Value**
k	Force coefficient	1.9×10^{-5} [N m²/A²]
I_m	Current eq. PM	13.4 [A]
L_m	Length of PM	3.0×10^{-3} [m]
μ_{rm}	Relative permeability of PM	1.09
m	One third of total mass	3.20 [kg]
l_e	Distance from origin to magnetic center	8.5×10^{-2} [m]
S	Pole face area	6.25×10^{-4} [m²]
N	Turn number of a coil	20 [turns]
R	Resistance of the winding at 30 [C°]	17 [Ω]
i_0	Current at nominal operational point (zero gravity bias due to PM)	0 [A]
x_0	Nominal gap length	7.56×10^{-3} [m]
K_{an}	Gap length stiffness	5.74×10^{3} [N/m]
K_{bn}	Current sensitivity	4.52 [N/A]
L_n	Nominal self-inductance of a coil	2.05×10^{-2} [H]

III. Linear Controller Design

The open loop system described in (7) is obviously unstable and an active control is needed for its stabilization[1]. Conventional approach to stabilize such a system is to apply linear controller-design method by using well-established design tools for linear systems[5].

Accordingly, a stabilizing controller design can be handled individually in dynamics of each single hybrid electromagnet. This approach has been has been named as "decentralized control method" in [3]. In spiteof some disadvantages discussed in [3], this decentralized control design method will be followed in this paper, since it is straightforward. Fundamental proportional and differential (PD) controllers for each hybrid electromagnet can stabilize the overall system around a specified linearization point. However, it can not satisfy the requirements of servo-control of its gap length, requested for a contactless object manipulation. Therefore, integral of tracking error is added to the feedback signal in order to enhance the capability of tracking the gap length, as well as to strengthen its robustness for unexpected external disturbance forces. State space technique is used in the following controller designs. Dynamic equations of the system is extended by adding the integral of the tracking error as an extra state variable in the state space form. Poles of the system are allocated to desired places by determining feedback matrix appropriately[3][5].

IV. Disturbance Observer

The application of state techniques requests full state information. Full state information should be acquired either by proper sensing devices or numerical state estimation based on observer theory. Since observers reconstruct unmeasured state variables from sensor signals when the observability is proven, they can be cost-effective solution. In a design of an active controller for an electromagnetic suspension system, measurement of gap length plays major role, and other state variables, *e.g.*, velocity of the length and the winding current can be readily estimated through the observer scheme.

A disturbance observer is an extension of well-known Luenberger observer. Effectiveness of the disturbance observer highly depends on accurate modeling of the dynamics of external force. In electromagnetic suspension system, one of the most crucial external disturbance is a stepwise change of its load force, namely, spontaneous payload variation. Zero-order disturbance observer emulating dynamics of stepwise disturbance has been commonly used for such studies. An original system matrix is slightly extended in order to include a zero-order disturbance dynamics. The observer gain vector providing faster dynamics than that of controller is determined by putting poles of the observer deeper into left hand side of the complex plain.

V. Fuzzy Model-Based Control Design

A fuzzy model-based control design is proposed and its application to electromagnetic suspension system are described in detail in this section.

A. Nonlinearity in a hybrid Electromagnet

The attractive force between hybrid electromagnet and ferromagnetic material is apparently nonlinear as linear explained in [1]. Linearized dynamics equivalent to (1) is the basis for a linear controller design in the form of stiffness coefficients as described in (4)-(6). Hence, the stiffness coefficients, K_a and K_b , representing the relationship between attractive force and the state variables, gap clearance, velocity and coil current play a crucial role in designing the controller. The variations of the linearized coefficients with the gap length and the coil current are summarized in Fig. 2 where the coefficients are normalized by the values at a nominal operating point[2][4]. It is seen from Fig. 2 that stiffness coefficients of the hybrid electromagnet exhibit considerable variations when the operating point slides on different parts of the operational domain. The linear controller has a fixed structure due to setting handled only a single operating point as seen from the flat plane in Fig. 2. Good control performance with wide variation of the gap length can not be expected to a linear controller for this reason: It may be possibly unstable. In order to overcome such a pitfall of a linear controller, fuzzy model-based control design approach employing distributed compensation principle[4] is being proposed in this paper.

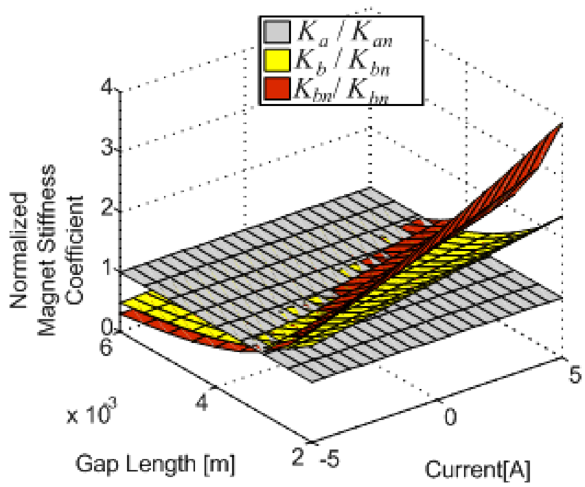


Fig. 2 Variation of normalized stiffness coefficients against the change of gap length and the coil current.

B. Dynamic Fuzzy Modeling of a Hybrid Electromagnet

Fuzzy modeling of a hybrid electromagnet has been executed through the following two steps;

i. derivation of local linear models, and
ii. proper combination of the local linear models by means of fuzzy interference algorithm.

In order to obtain local linear models within overall operational domain, admissible operating points for calculating data pairs of the gap length and coil current are determined. Choice of the admissible operating points requires a compromise having great importance in real time implementations. On the other hand, taking as many operating points as possible increases the capturing feature of the constructed fuzzy model in conjunction with proper fuzzy components, *e.g.*, membership functions. On the other hand, selection of a huge number of operating points results in complexity and will be a heavy computational load to a real-time processor. We have chosen only 5 admissible operating points to obtain linearized local dynamics for avoiding the problems. The straightforward behavior of our hybrid electromagnet under change of gap clearance command and external disturbance excitations provides intuitive guidelines for refining and also proper choice of the data pairs. For instance, if the gap clearance was ordered to get closer to ferromagnetic material, the coil current would take negative values, due to permanent magnet bias. By keeping in mind such cases, data pairs given in Table 2 have been used to construct local linear models.

The second phase of the dynamic fuzzy modeling is to determine structural components of the fuzzy interference algorithm, In the fuzzification stage, reference values of the gap clearance and the coil and the coil current are, mapped into corresponding membership function for fuzzy modeling. Membership functions employed in this study are illustrated in Fig. 3.

Table 2 Data pairs for local linear models

Gap length [m]	Coil current [A]	Model
1.5e-3	-3.77	Model 1
2.5e-3	-2.50	Model 2
4.5e-3	0.00	Model 3
6.0e-3	1.86	Model 4
7.0e-3	3.13	Model 5

Fuzzy model for hybrid electromagnet is derived formally as following manner by defining a rule base given in Table 3.

$$\Xi_1 \;:\text{IF}\;\; \underbrace{q_1 \text{ is } FS_1^1 \;\&\; q_2 \text{ is } FS_2^1}_{\text{antecedent part: gap length \& current}}$$

$$\text{THEN}\;\; \underbrace{\dot{\boldsymbol{x}} = A_1\boldsymbol{x} + B_1\boldsymbol{u} + \boldsymbol{D}_1 \;,\;\; y = C_1\boldsymbol{x}}_{\text{consequent part: local linear model}}$$

$$\vdots$$

$$\Xi_i \;:\text{IF}\;\; \underbrace{q_j \text{ is } FS_j^i \;\&\; q_j \text{ is } FS_j^i}_{\text{antecedent part: gap length \& current}}$$

$$\text{THEN}\;\; \underbrace{\dot{\boldsymbol{x}} = A_i\boldsymbol{x} + B_i\boldsymbol{u} + \boldsymbol{D}_i \;,\;\; y = C_i\boldsymbol{x}}_{\text{consequent part: local linear model}}$$

where $i = 1,2,\ldots,5$, and $j = 1,2$

$$\vdots$$

$$\Xi_5 \;:\text{IF}\;\; \underbrace{q_1 \text{ is } FS_1^5 \;\&\; q_2 \text{ is } FS_2^5}_{\text{antecedent part: gap length \& current}}$$

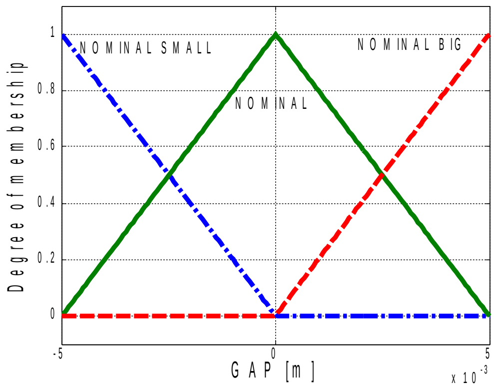
$$\text{THEN}\;\; \underbrace{\dot{\boldsymbol{x}} = A_5\boldsymbol{x} + B_5\boldsymbol{u} + \boldsymbol{D}_5 \;,\;\; y = C_5\boldsymbol{x}}_{\text{consequent part: local linear model}} \tag{12}$$

where q_1 and q_2 correspond to gap length and current, respectively, FS_j^i 's represent the fussy sets defined for gap length and current, Ξ_i stands for an associated fuzzy rule, and $\boldsymbol{D}_i$ is a term for disturbance effects. Let $\mu_j^i(x_j)$ be the membership of the fuzzy set FS_j^i . Subsequently, the state equation and the output equation synthesized by the fuzzy model of a hybrid electromagnet is expressed by using weighted average summation of local linear models as follows:

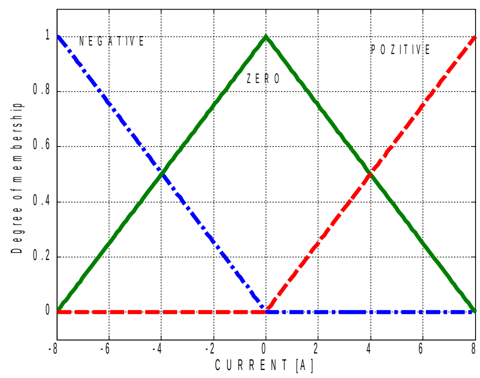
$$w^i = w^i(\boldsymbol{q}) = \prod_{j=1}^{2} \mu_j^i(q_j) \;\;,\qquad \alpha_i = \frac{w^i}{\sum_{i=1}^{5} w^i} \tag{13}$$

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{5} \alpha_i (A_i\boldsymbol{x} + B_i\boldsymbol{u}) \tag{14}$$

$$y = \sum_{i=1}^{5} \alpha_i (C_i\boldsymbol{x}) \tag{15}$$



(a) gap length



(b) coil current

Fig.3 Membership functions.

Table 3 Rule bases

RULE BASE		Gap length		
		Nominal Small	Nominal	Nominal Big
Current	Negative	Model 1	Model 4	Model 4
	Zero	Model 2	Model 3	Model 4
	Positive	Model 2	Model 2	Model5

C. Fuzzy Controller Design

After developing the fuzzy model of the system, fuzzy controller are designed for each one of the hybrid electromagnet by using parallel distributed control principle. The basic idea of parallel distributed system compensation principle is to design a linear controller for each one of the local linear models firstly, and to aggregate them by using fuzzy interference algorithm, secondly. The design of the fuzzy controller is described as follows:

$$\Xi_1 \; : \text{IF} \;\; q_1 \;\; \text{is} \;\; FS_1^1 \;\; \& \;\; q_2 \;\; \text{is} \;\; FS_2^1$$

$$\text{THEN} \quad \boldsymbol{u} = -K_1 \boldsymbol{x} + d_1 \qquad (16)$$

$$\vdots$$

$$\Xi_n \; : \text{IF} \;\; q_1 \;\; \text{is} \;\; FS_1^n \;\; \& \;\; q_2 \;\; \text{is} \;\; FS_2^n$$

$$\text{THEN} \quad \boldsymbol{u} = -K_n \boldsymbol{x} + d_n$$

$$n = 1, 2, \ldots, 5$$

$$\boldsymbol{u} = -\sum_{i=1}^{5} \alpha_i (K_i \boldsymbol{x} + d_i) \qquad (17)$$

Where the d_i correspond to offset bias at an associated linearization point. The membership functions and weighted average summation of the controller outputs same as in (16)-(17) are used in order to simplify the design process. The equation (17), *i.e*, total control action has nonlinear form and acts as a gain scheduler by changing the state feedback vector according to a change of gap length and coil current. Closed loop transfer function including state-space servo gap controller of a hybrid electromagnet has no zeros. Location of the poles for the nominal controller Model 3, the center of the five linear controllers, has been determined based on Kessler's canonical form[3]. The model 3 corresponding to the central nominal model has double roots in associated locations. These control scheme are graphically summarized in the block diagram in Fig. 4.

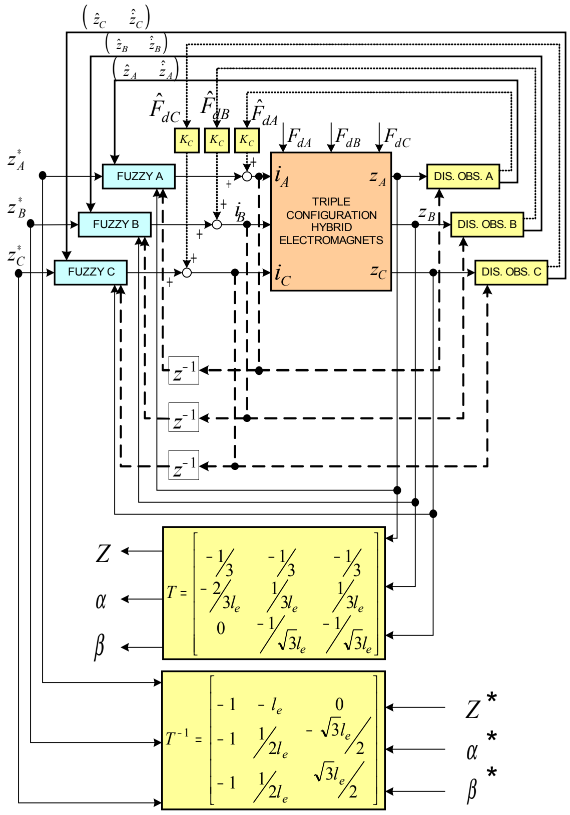


Fig. 4 Control block diagram of the Fuzzy model-based electromagnetic suspension.

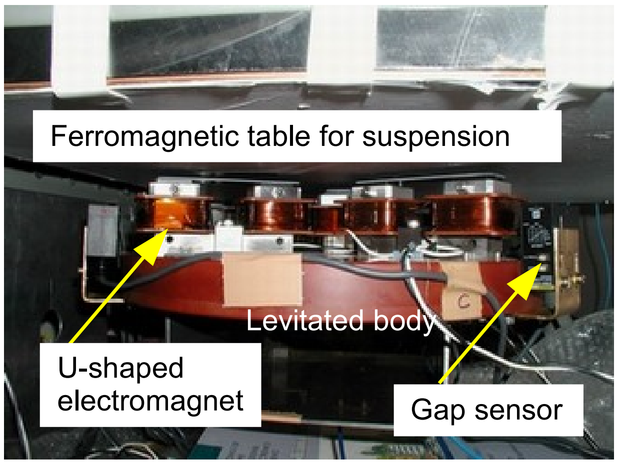


Fig. 5 Experimental test bench for magnetic levitation controls

VI. Experimental Results & Discussion

Fig. 5 shows our experimental levitated test bench. Implementation of the controllers were carried out in digital form by means of dSpace 1103 single processor multi function data acquisition board. Control algorithm were developed on MATLAB environment and uploaded to the real time processor board after compilation.

The sampling rate used in the experiments was 0.1msec. A nominal model, also taking place in fuzzy controllers as the Model 3, was applied in linear control experiments. Damping factor and equivalent time constant of the linear controller were set to 0.7 and 0.1 sec, respectively. Such setting is common in industrial applications and recommended by other researchers as well. Poles of the disturbance observers were set to 8 times faster than those of the controllers. Optical displacement sensors were used for sensing the change of the gap length. A Hall-type current sensor supplied by LEM-SA were used for monitoring the current flowing in the winding of the hybrid electromagnet.

Stepwise commands were simultaneously applied to each one of the global axes in order to examine response the system in terms of time domain criteria. Fig. 6 shows one of the results in such a case. Square wave reference command was applied to vertical axis in this experiment. The reference gap-command was set from 2mm to 8mm. Comparison of the results between linear and he proposed fuzzy controllers was shown in Fig. 6. The proposed Fuzzy model-based controller shows considerably better performance, especially, at a short gap length where the nonlinearity of the plant system is substantially strong.

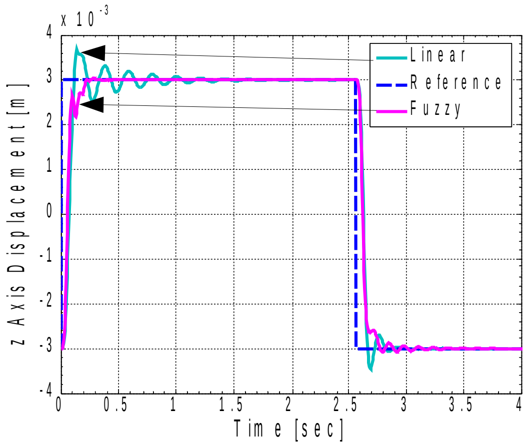


Fig. 6 Comparison of two controllers' time response to vertical displacement command

VII. Conclusions

This paper has described a systematic way of designing Fuzzy model-based controllers for a multiple degree-of-freedom electromagnetic suspension system. Stability of the feedback system is threatened when wide variation of the gap length is requested, due to nonlinearity of the electromagnetic force. However, the Fuzzy model-based controller readily embeds the nonlinearity and gives reliable gain tuning. Furthermore, you can directly extend linear control design principle by preparing basic controllers for the Fuzzy approach. A design of the Fuzzy model-based controller has been explained and demonstrated by keeping the straightforward extension from linear model to the nonlinear controller design in mind, in this paper. The advantage of the approach has been verified through an experiment. Consequently, our proposal;

A) lessens the overshoot,
B) eliminates the coupling effects between different motion direction,
C) extends stable gap-length operation range,
D) provides robustness to parameter changes,
E) presents a systematic and straightforward design of a nonlinear controller,
F) improves the levitation stability, and
G) provides possibly more comfortable magnetic levitation applications.

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