Practical Equivalent Circuit Model of Linear Induction Motors for Urban Transportation System Depending on Secondary Speed Based on Electromagnetic Analysis

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***Abstract*—A per-phase equivalent circuit model of linear induction motor (LIM) for transportation system is developed with circuit constants identified by parameter-fitting based on a present standard electromagnetic numerical calculations like finite element method (FEM), finite difference method (FDM) or actual testing result. In order to consider the longitudinal end-effect, circuit parameters are treated as functions of secondary speed. The new model based on this identification of equivalent circuit parameters can represent the energy flow in a LIM well and efficiently. Since the calculations based on the equivalent circuit are much lighter and more efficient than numerical approaches, this calculation technique may be useful for the design of LIM or a controller of LIM.**

***Index Terms*—Electromagnetic Analysis, End-effect, Equivalent Circuit, Linear Induction Motor**

I. Introduction

The progress of power electronics and control techniques brought good prospects to use linear motors in transport system. A linear induction motor (LIM) has been a suitable choice for railway application of both wheel-suspended and contact-less types, such as Linear Metro and HSST in Japan. The LIM has an advantage of simplicity in manufacturing, capability in applying direct force, independence of adhesiveness, movability over steep and turn of track, low cost of road maintenance and so on.

LIM's have inherent problems, which don't exist in conventional rotary induction motors because the magnetic motive force is not continuous in them. There is end-effect as major problem in LIM, which make the analysis, design and control of motors difficult and which have influence on the performance depending on its operation speed.

Numerical electromagnetic calculations like Finite Element Method (FEM) or Finite Difference Method (FDM) have been widely used for designs of electromagnetic machines and the design of its drive control system, and these numerical methods or actual measurements are needed for describing end-effect. On the other hand, for designing controller of a LIM, a direct application of the electromagnetic analysis or the actual measurement is restrictive because of expensive calculation time and manpower. It is convenient to use an equivalent circuit. Therefore, conventional induction motors' equivalent circuit is, however not directly applicable to the LIM without modification on the account of the end-effects.

An equivalent circuit of a LIM is identified from the electromagnetic analysis or actual testing result where the LIM's specific phenomena-end effect is taken into account. An equivalent circuit of a LIM was obtained as analytical formulation from the field analysis, which is Fourier transform method, space harmonic method or the other classical old theory of the LIM in the literatures [1][2][3]. Our method is based on a curve-fitting of an equivalent circuit of a LIM from numerical calculations. In addition, circuit constants are treated as function of the secondary speed determined by interpolation. The part of the numerical calculation, which is the information source for the curve-fitting, can be arbitrary substituted by measurement data. The proposed identification method is a generic method in practice in this sense.

II. Equivalent Circuit Model of a LIM as an Induction Motor

It is helpful to use an equivalent circuit for a simple modeling of a LIM. A per-phase equivalent circuit model of a conventional rotary induction motor is shown in Fig. 1.

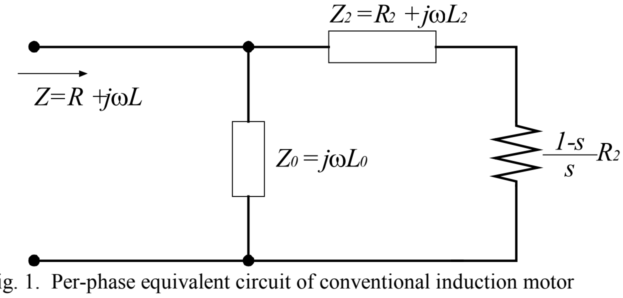

Fig. 1. Per-phase equivalent circuit of conventional induction motor

Since the core losses are neglected in this model, excitation impedance Z_0 consists of only reactance part. In addition, the primary winding impedance is eliminated because this impedance is ignored at in our numerical electromagnetic analysis, which is the base of identifying circuit constants. The primary winding resistance can be added afterwards if necessary.

Circuit constants are assume to be functions of speed because the end-effect depends mainly on its operational speed, in order to apply the frame of classical equivalent circuits of rotary induction motors to a LIM, in the region of actual operation point. These circuit constants are treated as "circuit parameters" and they are written as $L_0(v)$, $R_2(v)$, $L_2(v)$ as shown in Fig. 2.

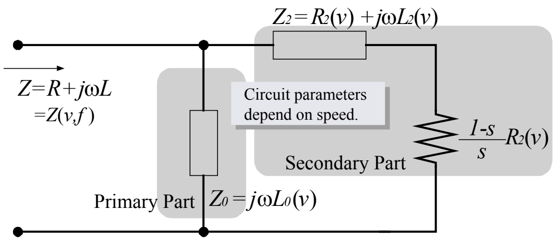

Fig. 2. Per-phase equivalent circuit extended for a LIM

If this conventional circuit frame is applicable to a LIM using only small modifications described above, then similar control theories to conventional rotary motors such as vector control can be applied to the LIM, which will contribute improving performance of a LIM-driven transport system.

III. Identification Method of Circuit Parameters

The absolute value of impedance $|Z|$ and the power factor $\cos\phi$ (= $R/|Z|$) obtained from equivalent circuits (Figs. 1 and 2) must be identical to those obtained from electromagnetic analysis for an equivalent circuit truly corresponding to an actual motor well. Identification methods of circuit parameters using the impedance calculated from electromagnetic analysis are described in this section.

A. General Identification Method

Equivalent circuit parameters are set from conditions that slip s is 0 and 1 in general rotary induction motor. The slip $s = 0$ is no-load state and $s = 1$ is blocked state. It is useful and practical for reduction of analysis or testing cost to use these two operating points.

Nevertheless, the both state $s = 1$ and $s = 0$ is far from actual operating conditions and especially in LIMs' case. This identification method leads to bad parameter identification for the LIM's.

B. New Identification Technique

1) Identification Steps

Circuit parameters can be assumed to change depending on operating conditions in the case of the LIM. In new identification method of circuit parameters, these are obtained from an electromagnetic analysis or the actual measurements. The identification flow is explained in Fig. 3.

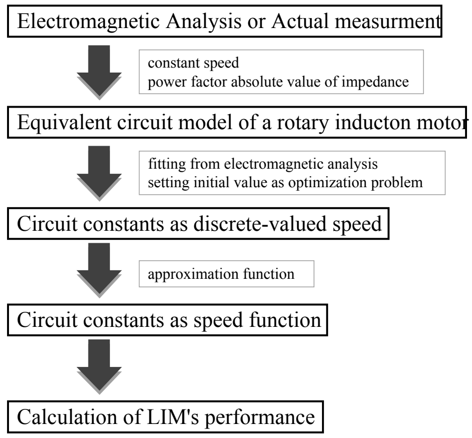

Fig. 3. New identification method of equivalent circuit for a LIM

Steps of the procedure of the identification are summarized as follows.

1. Choose some constant speed conditions to decide circuit parameters on the speed
2. Calculate absolute value of impedance $|Z|$ and power factor $\cos\phi$ under some of those speed conditions and some of points frequency f of power supply in actual operating region
3. Apply curve-fitting scheme described below to those calculation results and identify circuit constants under each speed condition
4. Obtain each circuit parameters as functions of speed by interpolation

2) Curve-fitting Scheme for Identification of Circuit Constants

Circuit parameters L_0, r_2, L_2 are identified based on curve-fitting scheme. The absolute value of impedance and power factor are represented by the function of frequency of power supply in the condition of fixed speed. These parameters are set by minimizing error between the curve from circuit parameters and analytical values at six points of frequency (or slip) under constant speed as shown in Fig. 4.

The $|Z_i|$ and $\cos\phi_i$ (i=1 to 6) are the absolute value of impedance and the power factor obtained from an equivalent circuit parameters, and $|Z_{mi}|$ and $\cos\phi_{mi}$ are those from electromagnetic analysis. The frequencies of power supply from f_1 to f_6 are chosen in the region of actual use of the LIM.

This scheme is an optimization problem which minimizes error between absolute value of impedance $|Z|$ and power factor $\cos\phi$ at the same time in six points of frequency conditions.

This problem can be formulated as follows:

$$\min F = \sum_{i=1}^{6}\left(\alpha\left(\frac{|Z_{mi}|-|Z_i|}{|Z_{mi}|}\right)^2 + (1-\alpha)\left(\frac{\cos\phi_{mi}-\cos\phi_i}{\cos\phi_{mi}}\right)^2\right) \quad (1)$$

where the α is weighting coefficient for converting multipurpose problem to mono-purpose problem and this

α is set to 0.5.

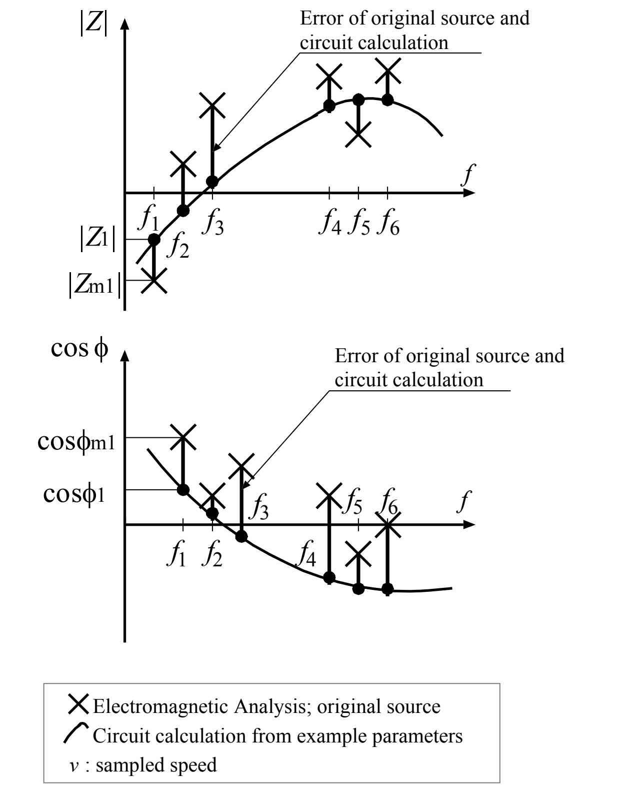

Fig. 4. Curve-fitting scheme of identification of circuit parameters under constant speed

The evaluation function F shown in (1) is minimized by using optimization toolbox of MATLAB. The selection of an initial value is significant for an appropriate search of the optimal value.

Sampled speeds for identification of circuit parameters are set to $v_1, v_2, \ldots, v_n$ where $v_1,< v_2 <, \ldots, < v_n$. Initial values of circuit parameters $L_0(v_1)$, $R_2(v_1)$ and $L_2(v_1)$ are determined from the same method of rotary induction motor, *i.e.*, those values are obtained by values when slip $s = 1$ and $s = 0$. Since the end-effects depend mainly on its operation speed, the effect is negligible at the lowest speed v_1.

For $v_2 <, \ldots,< v_m <, \ldots, < v_n$, circuit parameters $L_0(v_m)$, $R_2(v_m)$, $L_2(v_m)$ are identified from initial values which are successively $L_0(v_{m-1})$, $R_2(v_{m-1})$, $L_2(v_{m-1})$, by using (1).

IV. Analytical Model of a LIM

Electromagnetic numerical analysis like FEM and FDM or actual testing result is needed and applied for the purpose of identification of circuit constants. A LIM is analyzed using two-dimensional FDM(2D-FDM)[4], here LIM's transversal edge-effect is not taken into account in such 2D-calculation, but substantial consideration of its end-effect is included.

A. A 2D-Model of Linear Induction Motor

The Fig. 5 shows a model LIM based on HSST-200 Maglev vehicle, which is an EMS type Maglev vehicle [5]. This HSST-200 type is designed for the operation up to 200km/h.

The equivalent circuit model of this LIM is identified based on new identification technique described as previous section.

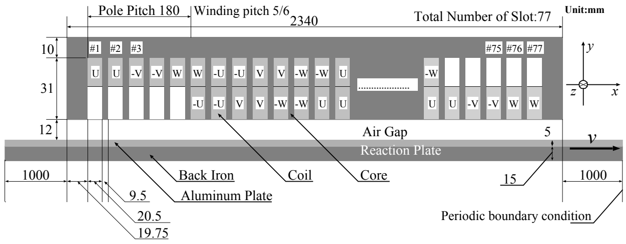

Fig. 5. Analytical model of a LIM based on HSST-200 vehicle

Physical values of z-axis direction are assumed constant and the width of core is set to 220mm. In addition, the change of air gap is neglected.

B. Other Design Values and Conditions

Other design values and conditions for FDM are summarized in Table I.

TABLE I
Other Data of The Analytical Model LIM

Conditions (Units)	Values
Nominal current (A)	400
Nominal frequency (Hz)	110
Slip frequency (Hz)	12.5(constant)
Turns of coil (Turns)	3
Magnetic resistance of primary core	$7.958*10^2$
Material of secondary conductor	Aluminum
Conductivity of secondary conductor (S/m)	$3.820*10^7$
Magnetic resistance of secondary conductor	$7.958*10^5$
Magnetic resistance of back iron	$7.958*10^2$

C. Conditions for Curve-fitting Scheme

Some constant speed v and set of frequency of power supply are summarized in Tables II and III respectively.

TABLE II
Choice of Constant Speed

Constant speed v_N	Value (km/h)
v_1	0.1
v_2	10
v_3	20
v_4	40
v_5	60
…	…
v_{12}	200

TABLE III
Choice of Frequency of Power Supply under Constant Speed

Frequency f	Slip freq. (Hz)	State	Value (Hz)
f_1	−13.5	RB*	$v/(2\tau)$−13.5
f_2	−12.5	RB	$v/(2\tau)$−12.5
f_3	−11.5	RB	$v/(2\tau)$−11.5
f_4	11.5	PR**	$v/(2\tau)$+11.5
f_5	12.5	PR	$v/(2\tau)$+12.5
f_6	13.5	PR	$v/(2\tau)$+13.5

Note
*RB: Regenerating Braking state
**PR: Power Running state
v: Constant speed such as $v_1, v_2, \ldots, v_{12}$
τ: Pole pitch (180mm)

V. Performance Characteristics of LIM

Performance characteristics of LIM calculated by using the equivalent circuit of Fig. 2 are compared with those from the electromagnetic analysis.

A. Frequency Characteristics of Impedance

Figs. 6 and 7 show frequency characteristics of the absolute value of impedance and power factor respectively under constant speed. Six circle points indicate FDM analysis result at reference frequency points from f_1 to f_6 shown in Table III for the curve-fitting under each speed. Moreover, continuous curve lines show characteristics calculated from the equivalent circuit identified at the speed.

Seeing these results, in almost all fitting points, the equivalent circuit can predict the characteristics calculated from using 2D-FDM with a few percentage of error in the region of actual use.

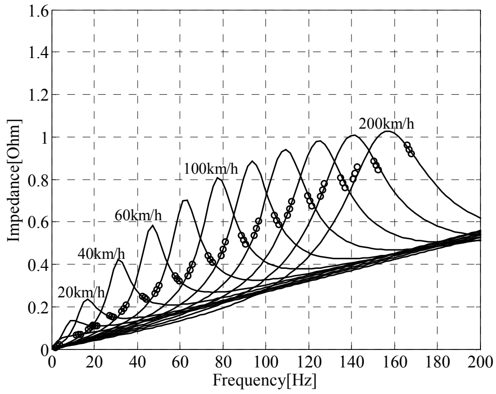

Fig. 6. Frequency characteristics of absolute value of impedance |*Z*|

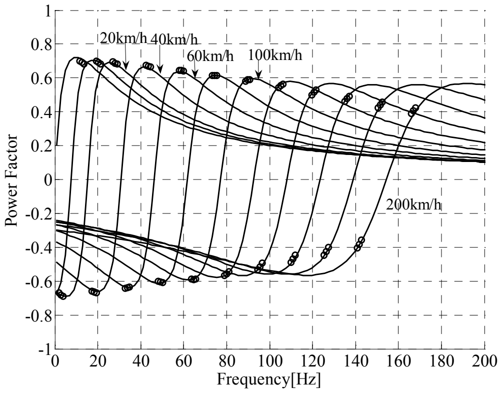

Fig. 7. Frequency characteristics of power factor cosϕ

B. Speed Characteristic of Circuit Parameters

Fig. 8 shows speed characteristics of each circuit parameters based on the curve-fitting scheme from 2D-FDM analyses. Circle points indicate parameters which is identified from original data of speed v_1, …, v_{12} and bold curves represent fitted approximate function of speed of each circuit parameters by interpolation using quadratic function.

In this Fig.6, R_0 becomes always zero because the core losses are neglected in this model. R_2 and L_0 correlate directly with operational speed on the other hand, L_2 does weakly with its speed.

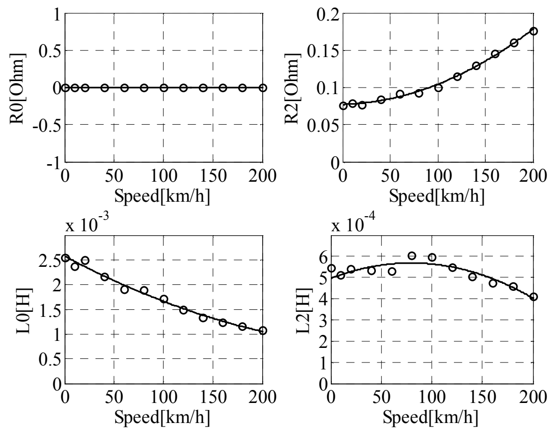

Fig. 8. Characteristics of circuit parameters depending on speed

The inductance L_0 decreases and the resistance R_2 increases with the increase of speed. It is qualitatively thought that the end-effect makes the thrust decrease as the increase of speed because the flux in air gap rises slowly and the interlinkage flux becomes small [6]. Thus, this phenomenon is expressed by decreasing the excitation inductance L_0. In addition, since the eddy current that denies the change of the flux comes significantly into existence in high-speed region, the secondary resistance R_2 increases.

C. Traction Force and Energy Flow

The characteristic of traction force is shown in Fig. 10 using circuit parameter in Fig. 8. This characteristic is the example of actual control of the LIM.

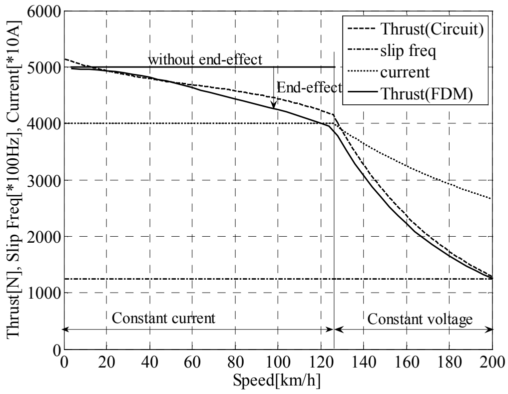

Fig. 9. Characteristics of traction force

The thrust characteristics of rotary induction motor must be constant in constant current mode, on other hand thrust decreases with the increase of speed even if this mode in LIM's. This equivalent circuit model can represent the characteristics including the end-effect, seeing this result of the traction force.

This traction force calculated by using equivalent circuit corresponds with that from FDM analysis within the error of 10%.

Fig. 10 shows the characteristics of input power and mechanical output and Fig. 11 does that of secondary copper loss calculated from the circuit and FDM analysis the same condition of the calculation of traction force for the propose of investigating the energy flow and the cause of the error.

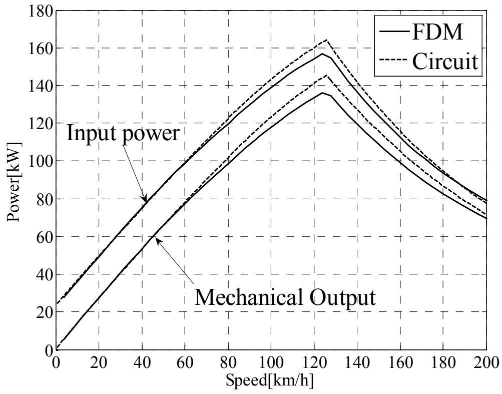

Fig. 10. Characteristics of input power and mechanical output

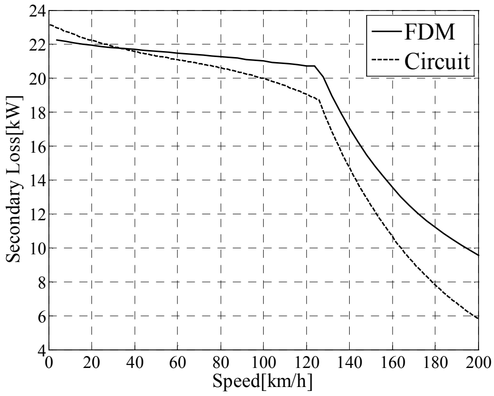

Fig. 11. Characteristics of secondary copper loss

Both input power and mechanical output from equivalent circuit become lager than that from FDM analysis in high-speed region. On another front, the secondary copper loss of the circuit is lower than that of FDM as the increase of speed.

Since the secondary copper loss evaluates lower than FDM though the end-effect is significant in high-speed, the increase of the loss caused by the end-effect is not taken into account and thrust calculated from the circuit is large, as this result.

Thus, simple extensions of an equivalent circuit model from rotary motors shown as Fig. 2 cannot express the end-effect perfectly.

VI. The Equivalent Circuit of LIM with Additional End-Effect Loss

A. Introduce of an Additional Term for the end-effect

The resistance R_c is added to the secondary part of the equivalent circuit of Fig. 2 in order to represent the loss of the end-effect as shown in Fig. 11.

The end-effect consists of a combination of entrance end-effect with exit end-effect. The entrance end-effect is mainly represented by using the circuit of a rotary type. Since the exit end-effect generates baking force and becomes energy consumption, the additional term R_c is introduced as shown in Fig.11.

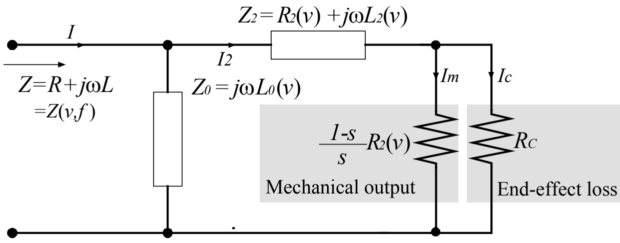

Fig. 11. Per-phase equivalent circuit of LIM including end-effect loss

B. Identification of the End-Effect Resistance

The resistance R_c is identified by the same method of the curve-fitting scheme. The selection of an initial value of R_c is significant for an appropriate search of the optimal value.

The initial value of R_c is determined by the exhaustive search in certain degree of region at the condition of sample speed v_{12} where the end-effect stands out comparatively.

The circuit parameters are identified without R_c once and then, circuit parameters introducing R_c are identified by the initial value when without R_c and the exhaustive search for R_c, using the curve-fitting scheme.

C. Performance Characteristics of LIM including End-Effect Resistance

Fig. 12 shows speed characteristics of each circuit parameters based on the curve-fitting scheme including the end-effect loss.

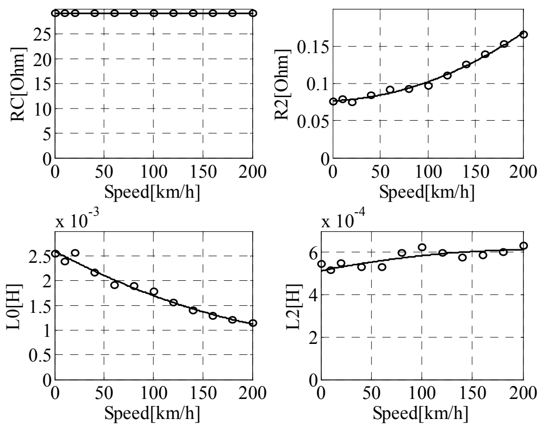

Fig. 12. Characteristics of circuit parameters depending on speed with the end-effect loss R_c

Speed characteristics of circuit parameters become the same phenomena described in Fig. 8. The end-effect resistance R_c is constant about 30 ohms and don't depend on secondary speed.

The characteristic of traction force is shown in Fig. 13, using this circuit parameters shown in Fig.12,. This traction force calculated by using this equivalent circuit corresponds with that from FDM analysis within the error of 1% in all speed regions.

Figs. 14 and 15 show the characteristics of power flows calculated from the circuit and FDM analysis the same condition of the calculation of traction force. Those energy flows make improvements, compared with the case without R_c. In addition, seeing the characteristic of the secondary losses shown in Fig. 14, the secondary copper loss represented from R_2, becomes similar result of the case of without R_c, on the other hand, the end-effect loss increases as the increase of secondary speed.

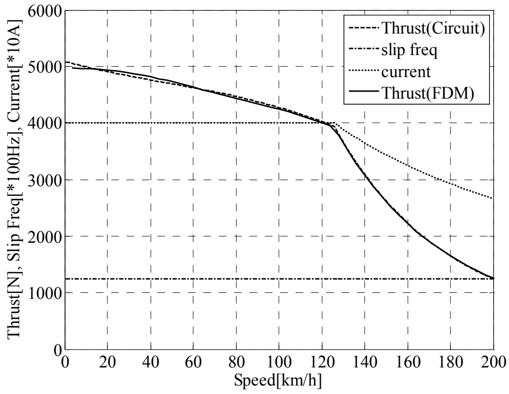

Fig. 13. Characteristics of traction force including end-effect loss

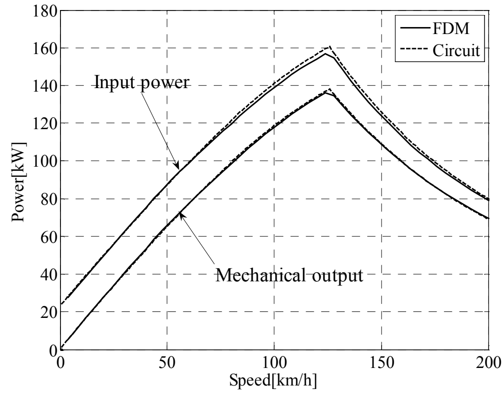

Fig. 14. Characteristics of input power and mechanical output

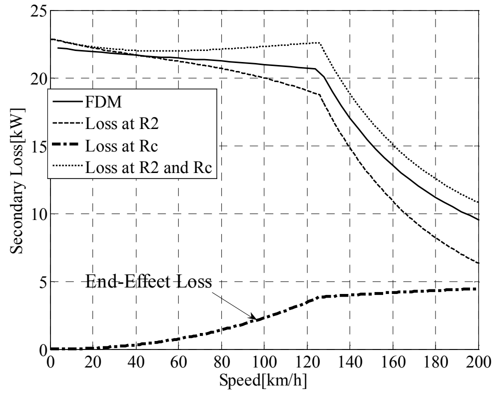

Fig. 15. Characteristics of secondary copper loss and the end-effect loss

Therefore, the increase of total secondary loss can be represented by introducing the end-effect loss resistance R_c, and this additional term has effective mean for considering the end-effect for an equivalent circuit model.

VII. CONCLUSIONS

New identification method of a per-phase equivalent circuit for a LIM is proposed based on an equivalent circuit of a rotary motor, using the curve-fitting of absolute value of impedances and power factors that is obtained from numerical electromagnetic analysis or actual measurement.

The circuit of rotary type with small modification assuming that there circuit parameters depend on secondary speed can represent the characteristics of a LIM within the error of 10% from source data which is calculated by using FDM analysis in the actual region of use.

In addition, new equivalent circuit of a LIM can calculate the characteristics corresponded to FDM analysis, introducing the additional term R_c that represents the loss of the exit-end-effect from the viewpoint of the energy flow.

This new equivalent circuit model of LIM and identification technique of the circuit parameters are practical for decreasing the calculation cost of numerical analysis and the manpower of actual measurements. Moreover, these are useful for realizing new controller of LIM and improving performance of a LIM-driven transport system.

Source data obtained from 3D numerical analysis or actual measurement will be applied and we will verify conformance with practical use from this time.

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