

# Fuzzy Control Applied to Stabilized Electromagnetic Suspension for Active Oscillation Suppression

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**Abstract** — The objective of this paper is to introduce a fuzzy model-based control design approach for an electromagnetic suspension stage on the purpose of active oscillation suppression. Furthermore, since the technique relies on state space control approach, rather than employing expensive velocity meter to obtain one of the state variable, gap clearance velocity, and at the same time to utilize the benefit of feed-forwarding the estimated disturbance through control loop, the disturbance observer design issue is discussed. The effectiveness of the proposed approach is demonstrated via comparative experimental results.

**Keywords** — fuzzy model-based control, oscillation suppression, electromagnetic suspension, disturbance observer.

## I. INTRODUCTION & MOTIVATION

Electromagnetic suspension technology (EMS), in which the attractive forces between electromagnets and ferromagnetic materials are utilized as suspension forces, has been successfully used in many industrial applications ranging from huge transportation systems in conjunction with linear motors to more advanced actuators for different purposes.[8]

Furthermore, active oscillation suppression systems, especially equipped with EMS mechanisms, have some inherited merits such as contact and dust-free operation and only electrical power source necessity. Thereby, they have a great potential to satisfy recent demands of industrial applications ranging from high precision control & measurement to clean room requirements. The principle structure of the active oscillation suppression system dealt with in this paper is depicted in Fig. 1.

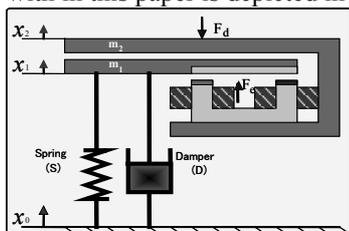


Fig. 1. Fundamental structure of active oscillation suppression system.

The system integrates permanent magnets into structure of electromagnet not only to overcome gravity bias but also to reduce the electromagnet size.[5]

Two external disturbance sources, namely direct disturbance force,  $F_d$ , and base excitations,  $x_0$ , cause undesired oscillations on upper mass,  $m_2$ , of the system. To obtain a more stable and realizable oscillation suppression system, employment of more than one electromagnet is indispensable. Hence, multiple degrees of freedom oscillation suppression capability can be achieved. To attain this goal, symmetrically arranged triple configuration of the U-shaped hybrid electromagnets has been proposed as seen in Fig. 2.[5][6]

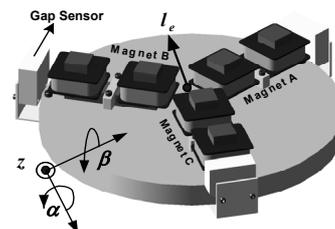


Fig. 2. Symmetrically arranged triple configuration of U-shaped hybrid electromagnets.

The organization of the paper as follows; firstly, controller design issue using linear state space approach will be introduced not only to provide a background for understanding of fuzzy counterpart but also to reveal its pitfalls. Then, fuzzy-model based control design and stability analysis based on numerical linear matrix inequality technique, LMI, will be briefly explained. To reconstruct the immeasurable state variables, which are needed for state feedback control, a zero order type disturbance observer design will be discussed. Subsequently, to verify the effectiveness of the proposed approach, comparative experimental will be released.

## II. MOTION DYNAMICS & CONTROLLER DESIGNS

### A. Motion Dynamics

Fundamental levitation model of a hybrid electromagnet in one-dimensional motion is depicted in Fig. 3.

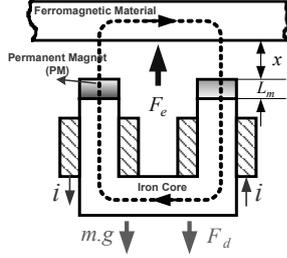


Fig. 3. Essential structure of a U-shaped hybrid electromagnet for one-dimensional motion case.

The developed attraction force of the hybrid electromagnet can be described by well-known magnetic circuit analysis techniques as;

$$F_e = k \left( \frac{i + I_m}{x + L_m / \mu_m} \right)^2 \quad (1)$$

Where,  $k$  is the force coefficient collecting magnet configuration parameters and in identification phases of actual plant, it can be obtained without any difficulty,  $i$  is the coil current,  $I_m$  is the equivalent permanent magnet current, it is also directly identified for actual plant,  $x$  is mechanical gap clearance,  $L_m$  is the length of electromagnet and  $\mu_m$  is the relative permeability of the permanent magnet.

By assuming that, each one of the electromagnet suspends one-third of the total mass,  $m_2$ , dynamic equation of the suspension including disturbance is expressed for magnet A by;

$$\frac{m_2}{3} \frac{d^2 x_A}{dt^2} = -k \left( \frac{i_A + I_{mA}}{x_A + L_{mA} / \mu_{mA}} \right)^2 + \frac{m_2 g}{3} + F_{dA} \quad (2)$$

Here,  $m_2$  denotes the total suspension mass and  $F_{dA}$  stands for outer disturbance on magnet A. Moreover, same equation can be derived for other two magnets, B & C, to describe completely the suspension dynamics.

### B. Linear Control Design

(2) is linearized around a specified operating point for tiny deviations in order to develop a linear controller as follows;

$$\frac{m_2}{3} \frac{d^2 \Delta x_A}{dt^2} = K_a \Delta x_A - K_b \Delta i_A + F_{dA} \quad (3)$$

$$K_a = - \left. \frac{\partial F_{eA}}{\partial x_A} \right|_{(i_{A0}, x_{A0})} \quad K_b = \left. \frac{\partial F_{eA}}{\partial i_A} \right|_{(i_{A0}, x_{A0})} \quad (4)$$

Where,  $K_a$  &  $K_b$  correspond to so called gap and current stiffness coefficients, respectively. Besides, they will have the same value for each one of the electromagnet since they are overall equivalent as mechanical structures.

The function of the electromagnet is to provide adjustable damping and stiffness to  $m_2$ , to suppress the oscillations imposed by external disturbance sources. Since, the system integrates permanents into the electromagnet configuration, zero power control based oscillation suppression can be a solution. However,

equivalence constraint of positive stiffness of mechanical elements, spring & damper, with negative stiffness property of the electromagnet makes it difficult to realize in practical applications.[4] To prevail over such a difficulty, gap clearance type control which is tracking passive elements' displacement has been proposed as a resolution.[7] The main idea of this approach is to solely control gap clearance of the electromagnet to provide the required damping and stiffness value without considering the mechanical dynamics. This understanding relaxes control and observer designs. Thereby, the remaining issue is high performance control of electromagnet over wide range of gap clearance operations.

The state space integral control algorithm has been favourably used in the design of linear controllers for magnetic suspension systems, since it can reduce the steady state error and as well as improve the robustness of the system against acting disturbances and uncertainties.[5] The gap clearance type control approach which is tracking passive elements' displacement can be derived in state space integral notation for one portion of Fig. 1 by;

$$\frac{d}{dt} \begin{bmatrix} \mathbf{x} \\ \int (x_1 - x_2) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & -\mathbf{BK}_i \\ A_i & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \int (x_1 - x_2) \end{bmatrix} + \begin{bmatrix} \mathbf{BK}_i \int (x_1 - x_0) \\ 0 \end{bmatrix} \quad (5)$$

Where,  $\mathbf{A}$  &  $\mathbf{B}$  represent system matrix of (3) in state space form,  $\mathbf{K}$  is feedback gain vector for original part of the state space equations and  $K_i$  stands for feedback gain of integral component.  $\mathbf{K}$  and  $K_i$  are determined by using one of the standard pole placement methods.

The simplest approach to achieve multiple degrees of freedom control of an electromagnetic suspension system comprised of several arrangements of electromagnets is to control solely each one of the electromagnet. This technique is called as decentralized control in literature and in this paper such an approach is followed.

### B. Fuzzy Control Design

It is a fact that the dynamics of the stage is apparently nonlinear, as seen from (1-2). The linear control design approaches try to solve this difficulty by employing linearization process as outlined previous section which has been yielding fixed controller structure. However, when the gap clearance takes the values far away from the specified operating point, ie.  $(i_0, x_0)$ , performance of the linear controller deteriorates and at last the instability occurs. Therefore, it is almost impossible to get operate the system for huge gap clearance variations by using linear control design techniques, even if it is intended to use, great care must be paid.[6][7] At this point, in contrast to linear controller design case, a global model using TSK (Takagi-Sugeno-Kang) fuzzy model-based control approach emerges as a candidate control design means by employing paralel distributed compensation principle in conjunction with linear control design methods.[2][3] It has some significant merits over the others.

- The well-known linear control design principles are utilized.
- Intuitive knowledge about the system can be readily contributed as a supplement tool to control design.

The idea behind the application of the TSK fuzzy model-based approach is that, TSK fuzzy model of the nonlinear system precisely represents the system's behaviour via aggregating many local affine & linear models. In electromagnetic suspension systems, linear local models can be derived easily via linearization process for admissible operating points. Then, TSK fuzzy inference algorithm comes into action by threading them to construct nonlinear model. TSK modelling is described formally by;

$$\begin{aligned} \Gamma 1 : & \underbrace{\text{IF } q_1 \text{ is } FS_1^1 \& q_2 \text{ is } FS_2^1}_{\substack{\text{AntecedentPart} \\ \text{Gap Clearance \& Current}}} \text{ THEN } \underbrace{\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_1 \mathbf{x} + \mathbf{B}_1 \mathbf{u} + \mathbf{d}_1 \\ y = \mathbf{C}_1 \mathbf{x} \end{cases}}_{\substack{\text{ConsequentPart} \\ \text{Corresponding Affine Model}}} \\ & \vdots \\ \Gamma n : & \text{IF } q_1 \text{ is } FS_1^n \& q_2 \text{ is } FS_2^n \text{ THEN } \begin{cases} \dot{\mathbf{x}} = \mathbf{A}_n \mathbf{x} + \mathbf{B}_n \mathbf{u} + \mathbf{d}_n \\ y = \mathbf{C}_n \mathbf{x} \end{cases} \end{aligned} \quad (6)$$

$i=1, 2, \dots, n$   
 $j=1, 2$

Where  $q_1$  &  $q_2$  correspond to gap clearance and current,  $FS$ s represent the fuzzy sets defined for gap clearance and current,  $\Gamma$  stands for associated fuzzy rule. Let  $\mu_j^i(x_j)$  be the membership of the fuzzy set  $FS_j^i$  and then;

$$\dot{\mathbf{x}} = \sum_{i=1}^n \alpha_i (\mathbf{A}_i \mathbf{x} + \mathbf{B}_i \mathbf{u}) \quad (7)$$

$$y = \sum_{i=1}^n \alpha_i (\mathbf{C}_i \mathbf{x}) \quad (8)$$

Designing a linear controller for each one of the affine local model and joining them together with fuzzy inference algorithm yield the basis of parallel distributed compensation principle. Namely, employment parallel distributed compensation technique provides a flexible means of determining the controller parameters according to change of the magnet operating point. Hence, appropriate parameterization of the controller can be handled simultaneously. Formally it is expressed by;

$$\Gamma 1 : \text{IF } q_1 \text{ is } FS_1^1 \& q_2 \text{ is } FS_2^1 \text{ THEN } \mathbf{u} = -\mathbf{K}_1 \mathbf{x} + d_1$$

$\vdots$

$$\Gamma n : \text{IF } q_1 \text{ is } FS_1^n \& q_2 \text{ is } FS_2^n \text{ THEN } \mathbf{u} = -\mathbf{K}_n \mathbf{x} + d_n$$

$$\mathbf{u} = -\sum_{i=1}^n \alpha_i (\mathbf{K}_i \mathbf{x} + d_i) \quad (10)$$

Consequently, (10) gives a nonlinear control action in accordance with change of the gap clearance and the coil current values.

Practical implementation of the outlined fuzzy controller is very much dependent number of rules defining the fuzzy model of the system. On the one hand, increase in the number of the rules improves capturing and generalizing property of fuzzy model and minimizes model-matching error between actual and fuzzy one. On the other hand, employment of as many as rules means

growing calculation complexity & cost and accordingly, heavy calculation burden for real time processor. Therefore, firstly, to obtain an admissible fuzzy controller some compromise must be done to select number of rules. Another fundamental issue is the shape of the membership functions. The job of the membership functions is to deduce fuzzy knowledge from the crisp ones. They have convexity features and can be described by well-known bell-shaped or Gaussian type distribution functions whose nonlinearity and as well as smoothes have significant effects on the model matching conditions of fuzzy implications. When the shape gets more complex, their mathematical manipulation in real time processor offers heavy calculation cost and fragile practical implementation. Therefore, in this research, we choosed 9 rules representing nonlinear dynamics of the system, (2), and determined membership function shapes as triangles and trapezoids as seen in Fig. 4.

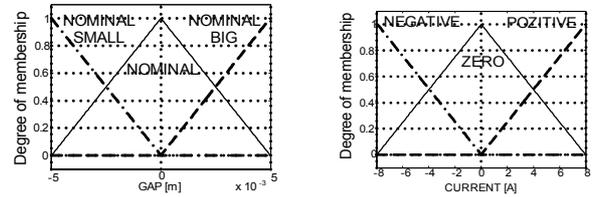


Fig. 4. Membership functions for gap clearance and coil current.

Eventually, the structure of the developed fuzzy system takes the form illustrated in Fig. 5.

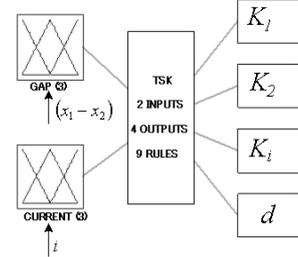


Fig. 5. Functional diagram of TSK fuzzy controller.

Stability assurance of linear control systems can be confirmed easily via checking locations of the closed-loop transfer function poles. Outlined fuzzy control algorithm employs linear models to determine the controller gains and thereby one may ask that, if each one of the local model has a stable controller, whether it is possible to obtain a full stable control system. The answer is no. Even each one of the local models has stable controller, their aggregation by fuzzy reasoning cannot provide stable operation.[2][3] Moreover, recently developed linear matrix inequality, LMI, based numerical techniques enable us to analyse the stability issue of over all system. The inspiration comes from well-known Lyapunov direct approach. Stability analysis issue is addressed by following theorem.[2]

**Theorem:** The equilibrium of the continuous fuzzy control system described by (8-11) is globally asymptotically stable if there exist a common positive definite matrix  $\mathbf{P}$  such that;

$$\begin{aligned} \mathbf{G}_{ii}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ii} < 0 \\ \left( \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2} \right)^T \mathbf{P} + \mathbf{P} \left( \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2} \right) \leq 0 \quad (11) \\ i < j \dots n \quad \text{s.t. } h_i \cap h_j = \emptyset \end{aligned}$$

Details of the proof are given in [2].

By using this theorem, stability of the system is confirmed for experimental investigations. The common  $\mathbf{P}$  matrix is obtained as

$$\mathbf{P} = \begin{bmatrix} 0.00410096 & 0.00002562 & -0.05141425 \\ 0.00002562 & 0.00000048 & -0.00004412 \\ -0.05141425 & -0.00004412 & 3.01081036 \end{bmatrix} \quad (12)$$

#### D. Disturbance Observer Design

Admissible operation of the state space controller relies on the availability of full state measurements. Our system is lack of velocity sensor, thereby, velocity must be reconstructed from the measurable state variables. One possibility is to obtain the velocities from gap sensor measurements via numerical derivative techniques. Yet, the noise coming out from the numerical derivative calculation is a serious practical difficulty and additionally, employment of low pass filter to reduce the noise level leads to time delay problem.[5][8]

However, since the system is observable, the velocity value can be reconstructed from a state observer. The classical observer suffers from the outer disturbances and parameter mismatches. Including the outer disturbance as a state variable with a known dynamics can improve the estimation quality and at the same time, if the defined disturbance dynamics matches the actual one, acting disturbance value can be easily observed and utilized for robust control purposes via simply feed-forwarding technique.[5][8] Here, to reconstruct velocity, zero order type disturbance observer design will be derived via extending system matrices by inclusion of disturbance value as state variable;

$$\frac{d}{dt} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{F}_d \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{F}_d \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{L} \\ L_d \end{bmatrix} \mathbf{C}(\mathbf{x} - \hat{\mathbf{x}}) \quad (13)$$

Where,  $\mathbf{L}$  &  $L_d$  correspond observer gain vector and can be determined one of the standard pole placement technique by giving faster response than that of controller.

The outlined control algorithms for fuzzy approach is depicted in Fig. 6. Furthermore to obtain generalized axes variables,  $(z, \alpha, \beta)$ , an axis transformation matrix,  $T$ , is derived via utilization of the geometrical relationships among displacements.

#### IV. EXPERIMENTAL RESULTS

For experimental verification purposes, an experimental test rig was constructed. A photo of the test bench is given in Fig. 7.

Controller & observer implementations were performed in digital forms. The sampling rate was 300  $\mu\text{sec}$ . Parameter set given in Table 1 was partially identified

from experimental test bench and utilized for practical implementations.

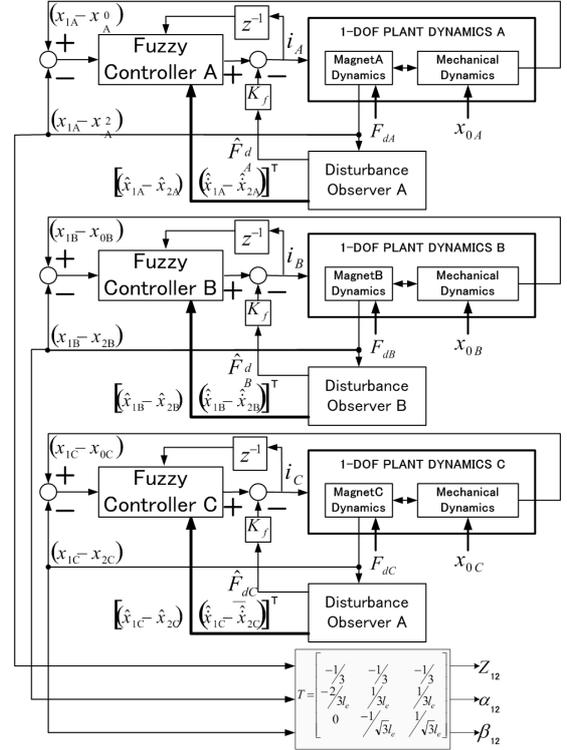


Fig. 6. Control block diagram for multiple degrees of freedom control.

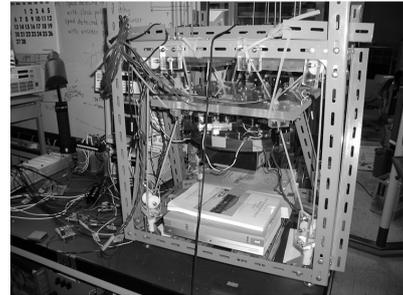


Fig. 7. A photo of experimental test bench.

TABLE I  
SPECIFICATIONS OF EXPERIMENTAL TEST BENCH

$k$	$2.01 \times 10^5$	$[\text{N m}^2/\text{A}^2]$	$M$	9.15	$[\text{kg}]$
$I_m$	12.4275	$[\text{A}]$	$i_0$	0	$[\text{A}]$
$L_m$	0.003	$[\text{m}]$	$z_0$	0.00734	$[\text{mm}]$
$\mu_m$	1.09		$g$	9.81	$[\text{kg}/(\text{m}/\text{sec}^2)]$
$\tau$	0.07	$[\text{sec}]$	$\gamma$	2.0	
$K_{b0}$	4.5053	$[\text{N}/\text{A}]$	$K_{a0}$	5398.3	$[\text{N}/\text{m}]$
$S$	2650	$[\text{N}/\text{m}]$	$D$	75	$[\text{N}/(\text{m}/\text{sec})]$

In fuzzy controller design, 9 attainable operation points were selected to derive local affine models. To obtain controller gains each one of the local affine models,

Kessler's canonical form was employed as pole location technique.

In the first experimental examination, gap clearance of the electromagnets commanded to change in sinusoidal form so as to result in direct disturbance force emulation on the masses of the system,  $m_1$  &  $m_2$ . Then immediately reference value of the gap clearance switched from sinusoidal waveform to oscillation suppression mode, namely, passive elements' displacements. Experimental results of this investigation are given in Fig. 8 a-b for both fuzzy and linear control cases, respectively. As seen from Fig. 8, the proposed gap clearance type control method following passive elements' displacements well damps the oscillations on  $m_2$ . Furthermore, fuzzy control approach shows less oscillatory behavior than linear counterpart especially at the instant of controller switching

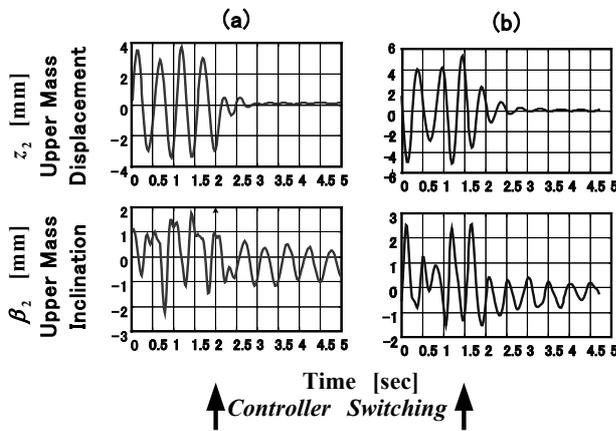


Fig. 8. Time response of emulated direct disturbance excitation on  $m_2$  (a) fuzzy control and (b) linear control.

Second experiment was conducted to investigate suppression characteristics of the oscillations induced by base excitations. In this case, an impulse base excitation was imposed to the supports of the suspension system. Experimental results are demonstrated in Fig. 9 a-b for both fuzzy and linear control cases, respectively. The results confirm that the proposed control policy, even works well for base excitations. Moreover, fuzzy control shows better rejection performance than that of linear one for impulse like excitations of the base.

## V. CONCLUSION

In this paper, an active oscillation suppression system using multiple configuration of stabilized hybrid electromagnets in conjunction with mechanical elements were introduced.

To resolve the oscillation suppression problem, the gap-length type control following displacement of passive elements was addressed.

To enable such a control approach for wide range of gap clearance operations, fuzzy model-based control

approach has been proposed. Design steps of the proposed technique were briefly explained and the stability analysis was performed in numerical sense.

To verify the effectiveness of the proposed fuzzy model-based algorithm over linear counterpart, the experimental results obtained from test bench have been exhibited. The experimental results reveal that the proposed fuzzy model-based control approach shows superior performance than that of linear counterpart for fundamental experimental tests and therefore it is quite promising for practical applications.

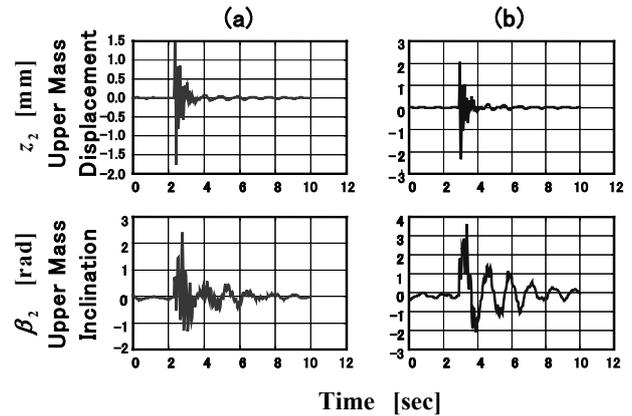


Fig. 9. Time response of impulse base excitation on  $m_2$  (a) fuzzy control and (b) linear control.

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