Flexible Nonlinear Stabilizing Control for Magnetic Levitation Based on A Fuzzy Algorithm for Safe and Comfortable Suspension of A Stage

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***Abstract* — This paper, introduces a fuzzy logic based flexible nonlinear stabilizing control design for a suspension stage on the purpose of safe and comfortable suspension realization. Furthermore, since the technique relies on state space control approach, rather than employing expensive velocity meter to obtain one of the state variable, gap clearance velocity, and at the same time to utilize the benefit of feed-forwarding the estimated disturbance through control loop, the disturbance design issue and its effects on suspension stability and safety is being discussed. The effectiveness of the proposed technique is demonstrated via comparative experimental results.**

***Keywords* — Fuzzy control, magnetic levitation, nonlinear control, disturbance observer.**

I. Introduction & Motivation

Electromagnetic suspension technology (EMS), in which the attractive forces between electromagnets and ferromagnetic materials are utilized as suspension forces, has been successfully used in many industrial applications ranging from huge transportation systems in conjunction with linear motors to more advanced actuators for different purposes. Furthermore, because of the availability of permanent magnets (PM), recently, there is a big tendency to merge PM's with conventional electromagnets so as to yield the remarkable advantages of so called hybrid electromagnets as; less power consumption, small sized power supply demand and reduction of magnet size.[1-4-5]

In the EMS systems, the U-shaped magnets are often used for generating the levitation force. The conventional U-shaped electromagnet, however, can only control one degree of freedom. It cannot construct a levitation system solely by itself. Multiple electromagnets must be arranged in a plane and be controlled simultaneously in order to construct a multi-degree of freedom levitation system. Recently, for different applications, many fascinating magnet configurations have emerged.[5]

Active control is indispensable part of the magnetic suspension systems, even for the simplest one, since the transfer function of the system has a root on the left half side of complex plane. Such inherent instability of electromagnets is still one of the most significant barriers, which is not allowing this technology to put into practical applications immediately and also threatening its safety. Besides, if wide range of the gap clearance operation is desired, which is becoming recently a significant demand, e.g. active vibration control, the problem of instability gets more difficult to solve, due to the apparent nonlinearity of the electromagnet.

In conventional manner, the stabilizing control design issue is solved simply by linearizing the nonlinear electromagnet motion equations around a specified equilibrium point for tiny deviations and then, designing a controller from the perspective of linear control design. However, as it has been briefly touched, the system shows obvious nonlinearity and thereby, if the deviations from the specified equilibrium point take huge values, the control performance degrades and consequently, instability occurs and safe suspension goal can not be achieved. The main reason of such a result is that the fixed structure of the linear controller. On the other hand, if the structure of the controller is adjusted smoothly then desired goal could be attained easily with required performance indices. Fortunately, fuzzy logic based control approach has been giving promising results by easily modifying and manipulating the linear design techniques, especially via TSK (**T**akagi-**S**ugeno-**K**ang) approach.[2]

In this paper, by keeping in mind the instability issue of electromagnet, fuzzy logic based nonlinear flexible control design approach for multi-degree of freedom electromagnetic suspension stage, comprised of triple configuration of electromagnets, is investigated. Furthermore, since TSK approach of fuzzy logic is being selected, owing to significant merits over others, state space techniques will be dealt with and therefore, the requirement of all states are provided via disturbance observer by also utilizing the benefit of feed-forwarding the estimated disturbance through control path. To reveal the effectiveness of the proposed algorithm over conventional linear one, comparative experimental results will be presented and discussed.

II. MOTION DYNAMICS OF ELECTROMAGNETIC SUSPENSION STAGE

Fundamental levitation model of a hybrid electromagnet in one-dimensional motion is depicted in Fig. 1

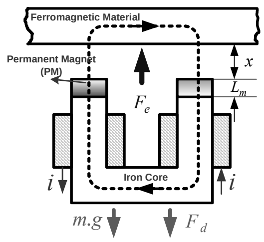

Fig. 1. Essential structure of a U-shaped hybrid electromagnet for one-dimensional motion case.

The developed attraction force of the hybrid electromagnet can be described easily by well-known magnetic circuit analysis techniques as;

$$F_e = k\left(\frac{i + I_m}{x + L_m / \mu_m}\right)^2 \quad (1)$$

Where, k is the force coefficient collecting magnet configuration parameters and in identification phases of actual plant, it can be obtained without any difficulty, i is the coil current, I_m is the equivalent permanent magnet current, it is also directly identified for actual plant, x is mechanical gap clearance, L_m is the length of electromagnet and μ_m is the relative permeability of the permanent magnet.

In this research, to obtain multi-degree of freedom electromagnetic suspension stage, three sets of U-shaped hybrid electromagnets are arranged in a geometrically symmetric manner on a plane as seen in Fig. 2.

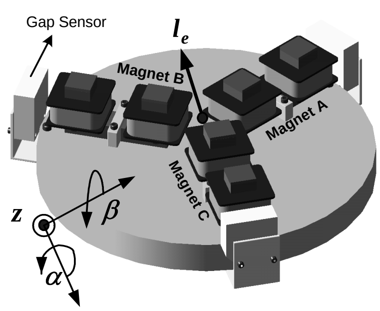

Fig. 2. Triple configuration of hybrid electromagnets on a plane.

When each one of the hybrid electromagnet suspends one-third of the total mass, dynamic equation of the motion including disturbance is expressed for magnet A by;

$$\frac{M}{3}\frac{d^2 x_A}{dt} = -k\left(\frac{i_A + I_{mA}}{x_A + L_{mA} / \mu_{mA}}\right)^2 + \frac{Mg}{3} + F_{dA} \quad (2)$$

Here, M denotes the total suspension mass and F_{dA} stands for outer disturbance on magnet A. Notice that subscript "$_A$" convention is used to relate associated variables & parameters to magnet A, additionally, the same equation can be derived for other two magnet, B & C, to describe completely the system dynamics.[3]

III. CONTROLLER & OBSERVER DESIGN

A. Linear Controller Design

The simplest approach to achieve the active control of the electromagnetic suspension stage illustarted in Fig. 2 is to control each one of the electromagnet solely. [3]

The nonlinear force eqaution, (1), is linearized around a specified equilibrium, ($i_{0,}$ x_0), for tiny deviations to lead linear control design as illustrated by;

$$k\left(\frac{i_{A0} + I_{mA}}{x_{A0} + L_{mA} / \mu_{mA}}\right)^2 = \frac{Mg}{3} \quad (3)$$

$$\frac{M}{3}\frac{d^2 \Delta x_A}{dt} = K_a \Delta x_A - K_b \Delta i_A + F_{dA} \quad (4)$$

$$K_a = -\left.\frac{\partial F_{eA}}{\partial x_A}\right|_{(i_{A0}, x_{A0})} \quad (5)$$

$$K_b = \left.\frac{\partial F_{eA}}{\partial i_A}\right|_{(i_{A0}, x_{A0})} \quad (6)$$

Where, K_a & K_b correspond the so called gap and current stiffness coefficients, respectively. Besides, they will have the same value for each one of the electromagnet since they are all equivalent as mechnaical structures. The linearized dynamic equation of the motion, (4), is written in the form of state space as;

$$\begin{bmatrix} \Delta\dot{z}_A \\ \Delta\ddot{z}_A \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{3K_a}{M} & 0 \end{bmatrix}\begin{bmatrix} \Delta z_A \\ \Delta\dot{z}_A \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{3K_b}{M} \end{bmatrix}\Delta i + \begin{bmatrix} 0 \\ \frac{3}{M} \end{bmatrix} F_{dA} \quad (7)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \Delta\dot{z}_A \\ \Delta z_A \end{bmatrix} \quad (8)$$

The state space integral control algorithm has been favourably used in the design of linear controllers for magnetic suspension systems, since it can reduce the steady state error and as well as improve the robustness of the system against acting disturbances and uncertanties.[3] The integral term is included into the system via extension of the states by;

$$\frac{d}{dt}\mathbf{x}_e = \mathbf{A}_e\mathbf{x}_e + \mathbf{B}_e\mathbf{u}_e + \mathbf{B}_{de}\mathbf{u}_{de} \quad (9)$$

$$\begin{bmatrix} \Delta\dot{z}_A \\ \Delta\ddot{z}_A \\ \Delta z_A \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{3K_a}{M} & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta z_A \\ \Delta\dot{z}_A \\ \int\Delta z_A \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{3K_b}{M} \\ 0 \end{bmatrix}\Delta i_A + \begin{bmatrix} 0 \\ \frac{3}{M} \\ 0 \end{bmatrix} F_{dA} \quad (10)$$

Having that each one of the state variables is available, the system poles are located on desired places by employing standard pole placement techniques.

$$\mathbf{u} = -\mathbf{K}\mathbf{x}_e \tag{11}$$

$$\Delta i_A = -\begin{bmatrix} K_{gA} & K_{vA} & K_{iA} \end{bmatrix} \begin{bmatrix} \Delta z_A \\ \Delta \dot{z}_A \\ \int \left(\Delta z_A - \Delta z_A^{*} \right) dt \end{bmatrix} \tag{12}$$

B. Fuzzy Controller Design

The dynamics of the suspension stage is apperantly nonlinear, as seen from (2). The linear control design approaches try to solve this difficulty by employing linearization process as outlined previous section which has been yielding fixed controller structure. However, when the gap clerance takes the values far away from the specified operating point, ($i_{0,}$ z_0), performance of the linear controller deteriorates and at last the system falls into instabilitiy. Such nonlinearity effect of the electromagnet is illustrated in Fig. 3 for magnet stiffness parameters, K_a & K_b. Therefore, it is almost impossible to get operate the system for huge gap clerance variations by using linear control design techniques, even, it is intended to use great care must be paid.[4]

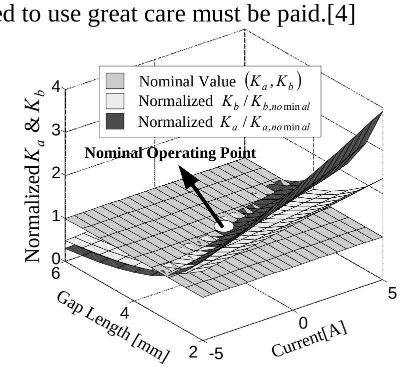

Fig. 3. Nominalized variations of magnet stiffness parameters, K_a & K_b , according to change of operating point.

At this moment, in contrast to linear controller design case, a global model using TSK fuzzy model-based controller design methodology offers a flexible nonlinear controller design approach by employing paralel distributed compensation principle in conjunction with linear control design techniques.[2-4] It has some significant merits over the others.

- The well-known linear control design principles are utilized.
- Intiutive knowledge about the system can be readily contributed as a supplement tool to control design.

The idea behind the application of the TSK fuzzy model-based approach is that, TSK fuzzy model of the nonlinear system preciseley represents the system's behaviour via agreggating many local linear models. In electromagnetic suspension systems, as outlined, linear local models can be derived easily via linearization process for predefined operation points. Then, TSK fuzzy inference algorithm comes into action by threading them to build up nonlinear model. TSK modelling is described formally by;

$$\begin{aligned} &\Gamma\mathbf{1}: \text{IF} \underbrace{q_1 \text{ is } FS_1^1 \ \& \ q_2 \text{ is } FS_2^1}_{\substack{\text{Antecedent Part} \\ \text{Gap Clearance \& Current}}} \text{ THEN } \underbrace{\begin{matrix} \dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u} + \mathbf{d}_1 \\ y = \mathbf{C}_1\mathbf{x} \end{matrix}}_{\substack{\text{Consequent Part} \\ \text{Corresponding Linear Model}}} \\ &\vdots \\ &\Gamma n: \text{IF } q_1 \text{ is } FS_1^n \ \& \ q_2 \text{ is } FS_2^n \text{ THEN } \begin{matrix} \dot{\mathbf{x}} = \mathbf{A}_n\mathbf{x} + \mathbf{B}_n\mathbf{u} + \mathbf{d}_n \\ y = \mathbf{C}_n\mathbf{x} \end{matrix} \end{aligned} \tag{13}$$

i=1, 2, …n
j=1, 2

Where q_1 & q_2 correspond to gap clearance and current, FS's represents the fuzzy sets defined for gap clerance and current, Γ stands for associated fuzzy rule. Let $\mu_j^i(x_j)$ be the membership of the fuzzy set FS_j^i and then;

$$w^i = w^i(\mathbf{q}) = \prod_{j=1}^{2} \mu_j^i(q_j) \tag{14}$$

$$\alpha_i = \frac{w^i}{\sum_{i=1}^{n} w^i} \tag{15}$$

$$\dot{\mathbf{x}} = \sum_{i=1}^{n} \alpha_i \left(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u} \right) \tag{16}$$

$$y = \sum_{i=1}^{n} \alpha_i \left(\mathbf{C}_i\mathbf{x} \right) \tag{17}$$

Designing a linear controller for each one of the linear local model and joining them together with fuzzy inference algorithm yield the basis of parallel distributed compensation principle. Namely, employment parallel distributed compensation technique provides a flexible means of determinig the controller parameters according to change of the magnet operating point. Hence, appropriate parameterization of the controller can be handled simultaneously. Formally is it expreesed by;

$$\begin{aligned} &\Gamma\mathbf{1}: \text{IF } q_1 \text{ is } FS_1^1 \ \& \ q_2 \text{ is } FS_2^1 \text{ THEN } \ \mathbf{u} = -\mathbf{K}_1\mathbf{x} + d_1 \\ &\vdots \\ &\Gamma n: \text{IF } q_1 \text{ is } FS_1^n \ \& \ q_2 \text{ is } FS_2^n \text{ THEN } \ \mathbf{u} = -\mathbf{K}_n\mathbf{x} + d_n \end{aligned} \tag{18}$$

$$\mathbf{u} = -\sum_{i=1}^{n} \alpha_i \left(\mathbf{K}_i\mathbf{x} + d_i \right) \tag{19}$$

Consequently, (19) gives a nonlinear control action in accordance with change of the gap clerance and the coil current values.

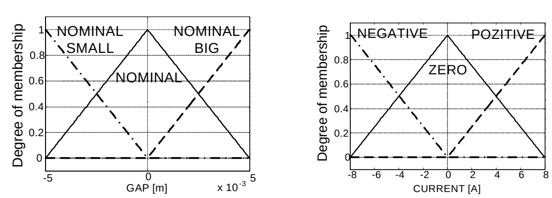

Fig. 4. Fuzzy membership functions of the gap clearance and coil currents.

Practical implementation of the outlined fuzzy controller is very much dependent number rules defining the fuzzy model of the system. On the one hand, increase in the number of the rules improves capturing and generalizing property of fuzzy model and minimizes model-matching error between actual and fuzzy one. On the other hand, employment of as many as rules means growing calculation complexity & cost and accordingly, heavy calculation burden for real time processor. Therefore, firstly, to obtain an admissible fuzzy controller, some compromise must be done to select number of rules. Another fundamental issue is shape of the membership functions. The role of the membership functions is to deduce fuzzy knowledge from the crisp ones. They have convexity features and can be described by well-known bell-shaped or Gaussian type distribution functions whose nonlinearity and as well as smoothes have significant effects on the model maching conditions of fuzzy implications. When the shape gets more complex, their mathematical manipulation in real time processor offers heavy calculation cost and fragile practical implementation. Therefore, in this research, we choosed 9 rules representing nonlinear dynamics of the system, (2), and determined membership function shapes as triangles and trapezoids as seen in Fig. 4.

Eventually, the structure of the developed fuzzy system takes the form illustrated in Fig. 5.

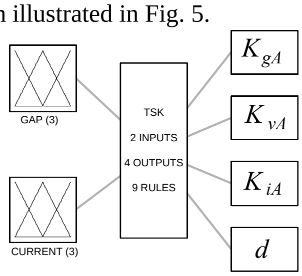

Fig. 5. The structure of TSK fuzzy controller.

C. Disturbance Observer Design

Admissible operation of the state space controller relies on the availability of full state measurements. Our system is lack of velocity sensor, thereby, velocity must be recostructed from the measurable state variables. One possibility is to acquire the velocities from gap sensor measurements via numerical derivative techniques. Yet, the noise arising from the numerical derivative calculation is a seious practial difficulty and additionally, employment of low pass filter to reduce the noise level leads to time delay problem.

However, since the system is observable, the velocity value can be reconstructed from a state observer. The classical observer suffers from the outer disturbances and parameter mismatches. Including the outer disturbance as state variable with a known dynamics can improve the estimation property and at the same time, if the defined disturbance dynamics matches the actual one, acting disturbance value can be easily observed and utilized for robust control purposes via simply feed-forwarding technique. Here, to reconstruct the gap clearance velocity, zero order type disturbance observer design will be derived via extending system matrices by inclusion of disturbance value as a state variable;

$$\begin{bmatrix} \Delta\dot{\hat{z}}_A \\ \Delta\ddot{\hat{z}}_A \\ \dot{\hat{F}}_{dA} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{3K_a}{M} & 0 & \dfrac{3}{M} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{z}_A \\ \Delta\dot{\hat{z}}_A \\ \hat{F}_{dA} \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{3K_b}{M} \\ 0 \end{bmatrix} \Delta i_A = \mathbf{A}_o \hat{\mathbf{x}}_o + \mathbf{B}_o \mathbf{u}_o \quad (20)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{z}_A & \Delta\dot{\hat{z}}_A & \hat{F}_{dA} \end{bmatrix}^T = \mathbf{C}_o \hat{\mathbf{x}} \quad (21)$$

$$\frac{d}{dt} \hat{\mathbf{x}}_o = \mathbf{A}_o \hat{\mathbf{x}}_o + \mathbf{B}_o \mathbf{u}_o + \mathbf{L}_o \left(\Delta z_A - \mathbf{C}_o \hat{\mathbf{x}}_o \right) \quad (22)$$

Where, the subscript "$_o$" represents extended state space matrices and vectors of the disturbance observer design equations, superscript " ^ " stands for observed state variables and **L** describes the observer gain vector. **L** is determined to give faster dynamic response than the controller.

The outlined control algorithms for both linear and fuzzy approaches are depicted in Fig. 6. & 7. Furthermore, to obtain generalized axes variables, (*z, α, β*), an axis transformation matrix, *T*, is derived via utilization of the geometrical relationships among gap clearance of the hybrid electromagnets.

IV. Experimental Results

The parameter data set given in Table 1. was utilized in experimental studies. Desired poles of the controllers and observers were determined by using Kessler's canonical form for both linear and fuzzy control cases. In Table 1., τ is the time constant of the Kessler's polynomial and γ is the stability index. The former one defines the rise time speed and in the meanwhile, stability index specifies wave shape of the gap clearance response, such as oscilatory or less oscilatory, for changing reference inputs and acting outer disturbance situations.

For fuzzy controller design, (2) was linearized over 9 achievable operating points. Accordingly, on account for the specified operating point, the plant parameters were obtained and controller gains were decided via help of Kessler's polinomial.

For experimental verification purposes, an experimental test bench, comprised of triple configuration of hybrid electromagnets, was assembled

TABLE I
Specifications of Experimental Test Bench

k	2.01×10^{-5}	[N m^2/A^2]	M	9.15	[kg]
I_m	12.4275	[A]	i_0	0	[A]
L_m	0.003	[m]	z_0	0.00734	[mm]
μ_m	1.09		g	9.81	[kg/(m/sec^2)]
τ	0.07	[sec]	γ	2.0	
K_{b0}	4.5053	[N/A]	K_{a0}	5398.3	[N/m]

as seen in Fig. 8. Controller design issues have been handled by employing Matlab package. Controller implementations were carried out in digital form via exploitation of dSpace 1103, single processor multi-function data acquisition board.

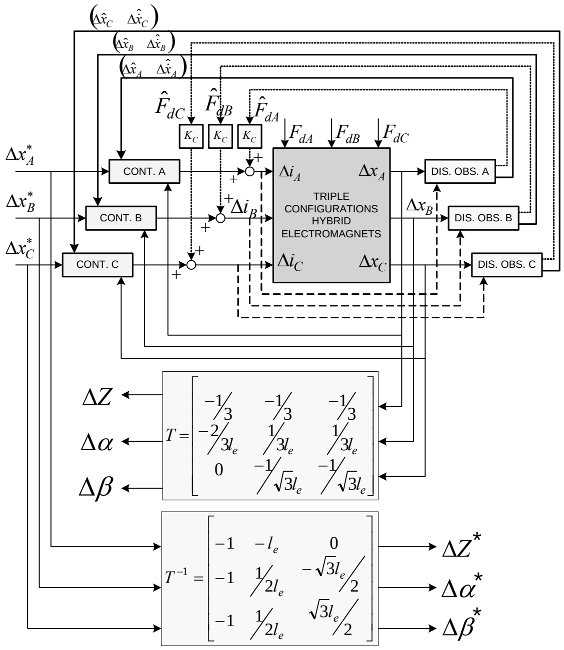

Fig. 6. Block diagram of the linear control system.

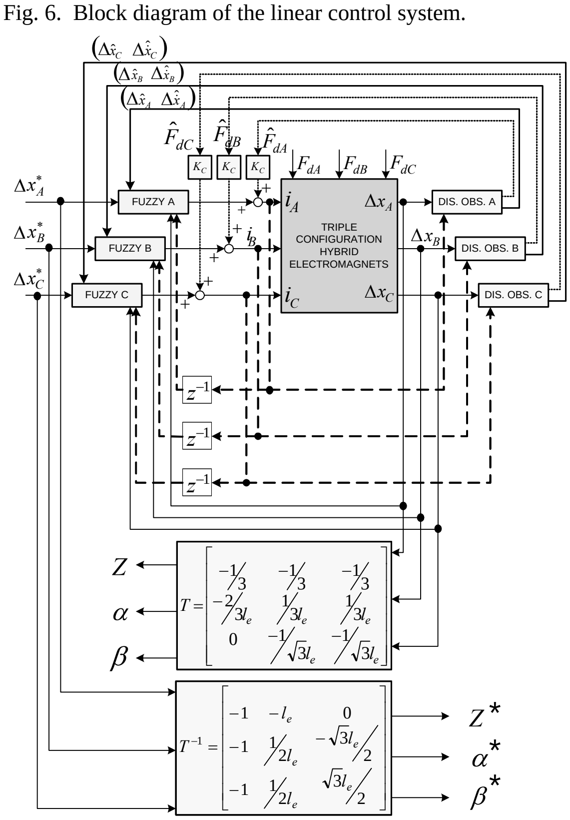

Fig. 7. Block diagram of the proposed fuzzy control system.

As a first experimental examination, the reference gap clerance change which was in stepwise form between ±*3 [mm]* were applied without feed-forwarding the estimated disturbance. The experimental results for this experiment are given in Fig. 9-10 for both absolute *z* and *β* axes.

Fig. 8. Photo of experimental test bench in levitation.

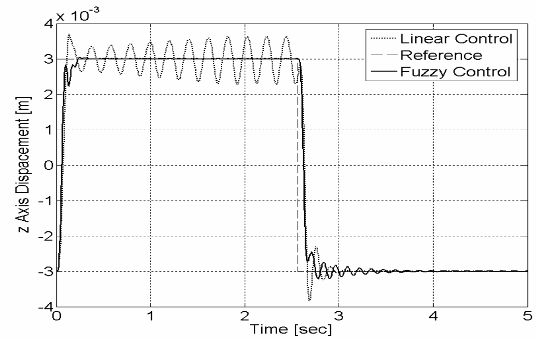

Fig. 9. . *z* axis time response for reference change without disturbance feed-forwarding.

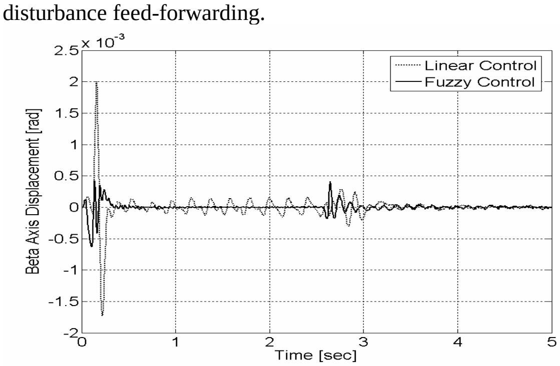

Fig. 10. *β* axis time response for reference change without disturbance feed-forwarding.

As seen from the Fig. 9-10, the proposed fuzzy controller design technique shows superiority over the linear counterpart for wide gap clerance change and reinforces the suspension safety. It verifies that if wide range of the gap clerance change is desired in operations, particularly for high performance control and as well as improved stability conditions, classical linear controller design approaches can not provide required control action by itself with fixed structure.

The second experimental test has been conducted again to observe the reference tracking performance by feed-forwardeding the estimated disturbance through control path for both linear and fuzzy control cases. Experimental results of this test are given in Fig. 11-12 for absolute *z* and *β* axes. As seen from Fig. 11-12, the estimated disturbance feed-forwarding can improve not only the system performance but also safety of suspension for both of the experimented control approaches. Interestingly, the performance of the linear controller was quite improved because of palnt nominalizing effect of the disturbance feed-frowarding process. Moreover, the linear controller design approach needs some more consideration to reduce overshoot and as well as lessen the oscilatory behaviour, while the fuzzy approach keeps on going its supremacy.

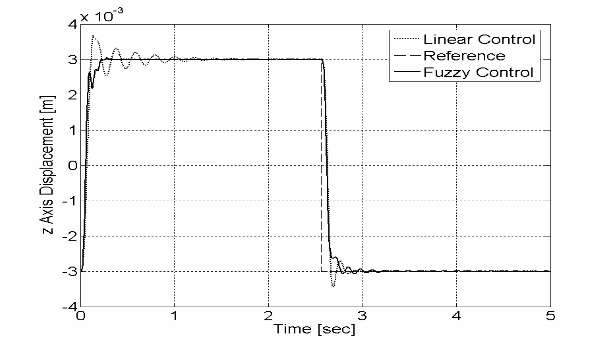

Fig. 11. z axis time response for reference change with disturbance feed-forwarding.

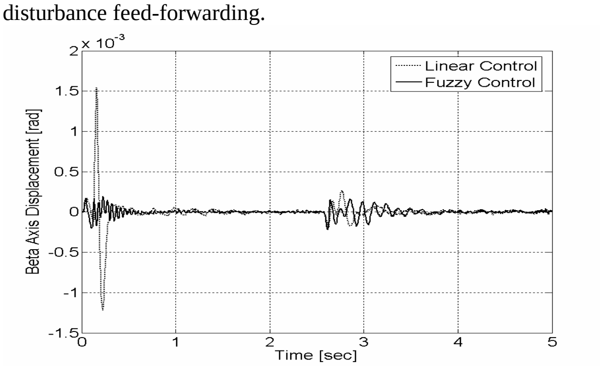

Fig. 12. β axis time response for reference change with disturbance feed-forwarding.

The last experiment was about the performance of the discussed controller structures in the case of outer disturbance excitation. To investigate such a case, *1.2 [kg]* pay-load was applied at *0.874 [sec]* to center of mass of the system for z axis, and the results depicted in Fig. 11-12 are obtained. From this results we can conlude that usage of the outlined fuzzy control approach improves both disturbance rejection performance and also suspension safety & comfort of the system.

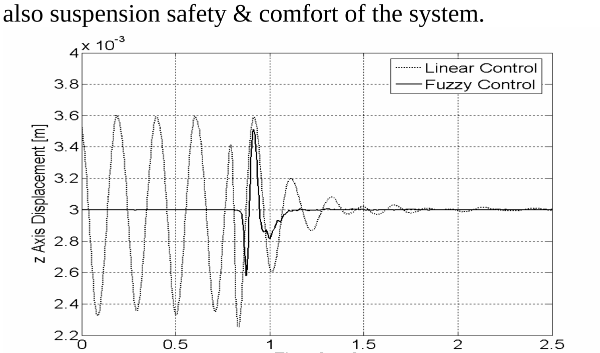

Fig. 13. z axis time response for *1.2 [kg]* pay-load disturbance acting @ *0.874 [sec]* without disturbance feed-forwarding.

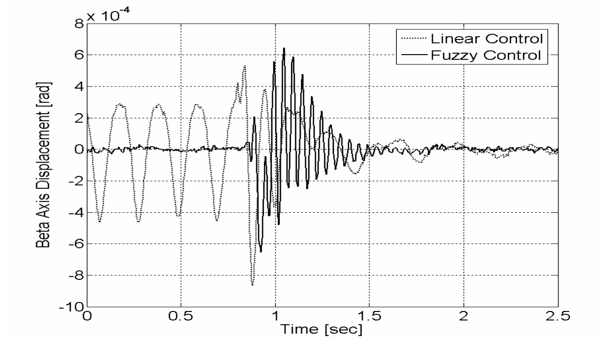

Fig. 14. β axis time response for *1.2 [kg]* pay-load disturbance acting @ *0.874 [sec]* without disturbance feed-forwarding.

V. Conclusion

In this paper, to extend the operating range, improve the robustness and at same time satisfy safety & comfort of an electromagnetic suspension stage, comprised of triple configuration of electromagnets, a fuzzy logic based flexible nonlinear stabilizing control is proposed. Furthermore, to reconstruct the immeasurable gap clearance velocity and to employ the benefit of disturbance estimation, zero order type disturbance observer design issue is described and verification & comparative studies over both fuzzy and conventional linear one have been demostrated through experimental results. The proposed fuzzy control technique, surely, shows superiority over conventional one. Also, employment of the disturbance observer considerably improves the performance of the both control design methodology. However, for wide range of gap clearance operations, it is still indispensable to employ fuzzy control with and/or without disturbance feed-forwarding for the safety and comfort of the subject matter suspension stage.

References

[1] E. P. Furlani, *Permanent Magnet and Electromechanical Device*, Academic Press, 2001.

[2] S. K. Hong and R. Langari, “Robust Fuzzy Control of Magnetic Bearing System Subject to Harmonic Disturbances”, *IEEE Transactions on Control Systems Technology*, Vol.08, No.02, pp.366-371, 2000.

[3] Y. Makino, *Six Degrees of Freedom Control Through Three Hybrid Electromagnets and Three Linear Induction Motor for Two Dimensional Conveyance System*, Master Thesis, The University of Tokyo, 2004.

[4] K. Erkan and T. Koseki, “Disturbance Identification Based on Fuzzy Logic for Active Vibration Control Using U Type Hybrid Electromagnet”, *The IEE Japan Tech. Meeting on Linear Drives,* LD-04-84-96, Dec. 2004.

[5] J. Liu and T. Koseki, “3 Degrees of Freedom Semi-Zero-Power Maglev Scheme for Two-Dimensional Linear Motor”, *Proceeding of Linear Drives for Industry Applications*, pp.30-35, October 2001.