3 Degrees of Freedom Fuzzy Model-Based Nonlinear Control of Triple Configuration of U-Type Hybrid Electromagnets

Kadir Erkan*† and Takafumi Koseki*††

*The University of Tokyo, 7-3-1, Hongo, Bunkyou-Ku, Tokyo 113-8656, JAPAN,
Phone +81-3-5841-6676/6791, Facsimile +81-3-5800-5988
Email: †erkan@koseki.t.u-tokyo.ac.jp, ††koseki@koseki.t.u-tokyo.ac.jp

Abstract Since conventional linear controller approaches have fixed structure, they are not suitable for high performance control of electromagnetic suspension systems especially for wide range of gap clerance operations. The main reason of the such an incosistency is nonlinear force characteristics of the electromagnets. In this paper, to deal with such an issue, fuzzy model-based control design approach is proposed. The proposed fuzzy control algorithm readily embeds nonlinear features of the electromagnets into design process of controller by employing linear controller design principles. From this point view, it is a sligth extension of linear controller approaches except changing for controller parameters. Furthermore, to recostruct the immeasurable gap clearance velocity value from gap length sensor measurement and also to use the benefit of disturbance compensation, zero order disturbance observer design issue is discussed via experimental studies for a magnetic suspension system comprised of triple configuration hybrid electromagnets.

Keywords : Fuzzy model-based control, 3 degrees of freedom control, magnetic levitation, disturbance observer.

1. INTRODUCTION & MOTIVATION

Recently, because of the availability of permanent magnets, there is a big tendency to combine conventional electromagnets with permanent magnets to contruct so called hybrid ones yielding less power consumption and as well as reduced magnet size. Furthermore, multiple degrees of freedom controlled hybrid electromagnetic suspension systems have been getting great importance in the applications ranging from magnetic levatiation based transporation systems to more complex actuators.[1]

In this research, triple configuration of the U-type hybrid electromagnets is preferred to contruct three degrees of freedom control capability by means of electromagnetic suspension technology as illustrated in Fig. 1.

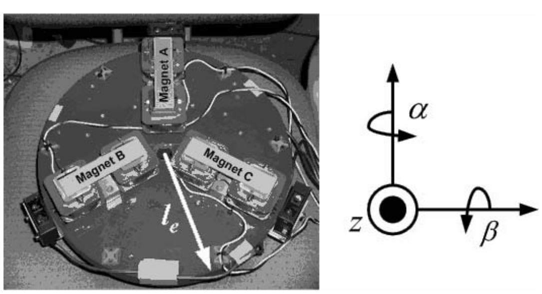

Fig. 1. Triple configuration of hybrid electromagnets.

For this system, high performance three degrees of freedom control design is rather challenging task especially for wide change of gap clerance of electromagnets, owing to the apperant nonlinearity characteristic of developed electromagnetic force. Conventional linear control techniques deal with this problem by linearizing the developed electromagnetic force characteristic a specified equilibrium point for tiny deviations. On the other hand, for advanced applications, it is required to adjust gap clearance in broad ranges. Since, the conventional linear control techniques offer fixed controller structures, for moderate alteration of the gap clearance from the specified operation point, the control performance of conventional linear controller degrades and even results in instability. Fortunately, TSK (**T**akagi-**S**ugeno-**K**ang) fuzzy model based control approach overcomes the nonlinearity issue by preserving linear controller design easiness via employing smooth transition among the specified linear controllers according to change of operating point.[2]

In this paper, to obtain high performance three degrees of freedom control performance, TSK fuzzy model based control approach is selected as the candidate. To verify the effectiveness of the selected technique over the conventional state space integral control approach, experimental studies have been conducted. Furthermore, because of the unavailability of the gap clerance velocity sensors, to reconstruct the velocity signal from measurements of gap clearance sensors and also compensate external disturbance via estimated disturbance feed-forwarding, disturbance observer design issues have been taken into consideration.

2. DYNAMICS OF THE SYSTEM

For a U-type hybrid electromagnet the developed electromagnetic force can described by employing well-known magnetic circuit techniques as following,

$$F_e = k\left(\frac{i + I_m}{z + L_m / \mu_m}\right)^2 \qquad (1)$$

Where, k is the force coefficient collecting magnet configuration parameters, i is the coil current, I_m is the equivalent permanent magnet current, z is mechanical gap clerance, L_m is the length of electromagnet and μ_m is the relative permeability of the permanent magnet. When the each one of the hybrid electromagnet suspends one-third of the total mass, dynamic equation of the motions including disturbance is stated for magnet A by;

$$\frac{M}{3}\frac{d^2 z_A}{dt} = -k\left(\frac{i_A + I_{mA}}{z_A + L_{mA}/\mu_{mA}}\right)^2 + \frac{Mg}{3} + F_{dA} \qquad (2)$$

Here, M denotes the total suspension mass and F_d stands for outer disturbance. Notice that subscript "$_A$" convention is used to relate associated variables and parameters to magnet A, additionally, the same equation can be derived for other two magnet, B & C, to express completely the system dynamics.

3. CONTROLLER & OBSERVER DESIGN

3.1 LINEAR CONTROLLER DESIGN

The nonlinear force eqaution (2) is linearized around a specified equilibrium, ($i_{0,}$ z_0), for tiny deviations to lead linear control design as illustrated by;

$$k\left(\frac{i_{A0} + I_{mA}}{z_{A0} + L_{mA}/\mu_{mA}}\right)^2 = \frac{Mg}{3} \qquad (3)$$

$$\frac{M}{3}\frac{d^2 z_A}{dt} = K_a \Delta z_A - K_b \Delta i_A + F_{dA} \qquad (4)$$

$$K_a = -\left.\frac{\partial F_{eA}}{\partial z_A}\right|_{(i_{A0}, z_{A0})} \qquad (5)$$

$$K_b = \left.\frac{\partial F_{eA}}{\partial z_A}\right|_{(i_{A0}, z_{A0})} \qquad (6)$$

Where, K_a & K_b correspond the so called gap and current stiffness coefficients, respectively. Besides, they will have the same value for each one of the electromagnet since they are all equivalent as mechnaical structures. The linearized dynamic equation of the motion, (4), is written in the form of state space as;

$$\frac{d}{dt}\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{B}_d \mathbf{u}_d \qquad (7)$$

$$\begin{bmatrix} \Delta\dot{z}_A \\ \Delta\ddot{z}_A \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{3K_a}{M} & 0 \end{bmatrix}\begin{bmatrix} \Delta z_A \\ \Delta\dot{z}_A \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{3K_b}{M} \end{bmatrix}\Delta i + \begin{bmatrix} 0 \\ \frac{3}{M} \end{bmatrix} F_{dA} \qquad (8)$$

$$y = \mathbf{Cx} \qquad (9)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \Delta\dot{z}_A \\ \Delta z_A \end{bmatrix} \qquad (10)$$

The state space integral control algorithm has been favourably used in the design of linear controllers for magnetic suspension systems since it can reduce the steady state error and as well as improve the robustness of the system against acting disturbances and uncertanties.[3] The integral term is included into the system via extension of the states by;

$$\frac{d}{dt}\mathbf{x}_e = \mathbf{A}_e \mathbf{x}_e + \mathbf{B}_e \mathbf{u}_e + \mathbf{B}_{de} \mathbf{u}_{de} \qquad (11)$$

$$\begin{bmatrix} \Delta\dot{z}_A \\ \Delta\ddot{z}_A \\ \Delta z_A \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{3K_a}{M} & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta z_A \\ \Delta\dot{z}_A \\ \int \Delta z_A \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{3K_b}{M} \\ 0 \end{bmatrix}\Delta i_A + \begin{bmatrix} 0 \\ \frac{3}{M} \\ 0 \end{bmatrix} F_{dA} \qquad (12)$$

Having that each one of the state variables is available, the system poles are located desired places by employing pole placement techniques.

$$\mathbf{u} = -\mathbf{K}\mathbf{x}_e \qquad (13)$$

$$\Delta i_A = -\begin{bmatrix} K_{gA} & K_{vA} & K_{iA} \end{bmatrix}\begin{bmatrix} \Delta z_A \\ \Delta\dot{z}_A \\ \int\left(\Delta z_A - \Delta z_A^{\ *}\right)dt \end{bmatrix} \qquad (14)$$

3.2 FUZZY CONTROLLER DESIGN

The dynamics of the system is apperantly nonlinear, as seen from (2). The linear control design approaches try to solve this issue by employing linearization process as outlined previous section which has been yielding fixed controller structure. However, when the gap clerance takes the values far away from the specified operating point, ($i_{0,}$ z_0), performance of the linear controller deteriorates and at last the system falls into instabilitiy. Therefore, it is almost impossible to get operate the system for huge gap clerance variations by using linear control design techniques. At this point, in contrast to linear controller design case, a global model using TSK fuzzy model-based control approach emerges as a candidate control design means by employing paralel distributed compensation principle in conjunction with linear control design methods.[4] It has some significant merits over the others.

- The well-known linear control design principles are utilized.
- Intiutive knowledge about the system can be readily contributed as a supplement to control design.

The idea behind the application of the TSK fuzzy model-based approach is that, TSK fuzzy model of the nonlinear system preciseley represents the system's behaviour via agreggating many loal linear models. In electromagnetic suspension systems, as outlined, linear local models can be derived easily via linearization process for tentative opreration points. Then, TSK fuzzy inference algorithm comes into action by threading them to construct nonlinear model. TSK modelling is described formally by;

$$\begin{aligned}
&\Gamma\mathbf{1}: \text{IF} \underbrace{q_1 \text{ is } FS_1^1 \ \& \ q_2 \text{ is } FS_2^1}_{\substack{\text{Antecedent Part} \\ \text{Gap Clearance \& Current}}} \text{THEN} \underbrace{\begin{matrix} \dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u} + \mathbf{d}_1 \\ y = \mathbf{C}_1\mathbf{x} \end{matrix}}_{\substack{\text{Consequent Part} \\ \text{Correspondng Linear Model}}} \\
&\vdots \\
&\Gamma n: \text{IF} \, q_1 \text{ is } FS_1^n \ \& \ q_2 \text{ is } FS_2^n \text{ THEN } \begin{matrix} \dot{\mathbf{x}} = \mathbf{A}_n\mathbf{x} + \mathbf{B}_n\mathbf{u} + \mathbf{d}_n \\ y = \mathbf{C}_n\mathbf{x} \end{matrix}
\end{aligned} \qquad (15)$$

i=1, 2, …n
j=1, 2

Where q_1 & q_2 correspond to gap clearance and current, FS's stand for fuzzy sets defined for gap clearance and current, Γ represents associated fuzzy rule. Let $\mu_j^i(x_j)$ be the membership of the fuzzy set FS_j^i and then;

$$w^i = w^i(\mathbf{q}) = \prod_{j=1}^{2} \mu_j^i(q_j) \qquad (16)$$

$$\alpha_i = \frac{w^i}{\sum_{i=1}^{n} w^i} \qquad (17)$$

$$\dot{\mathbf{x}} = \sum_{i=1}^{n} \alpha_i \left(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u}\right) \qquad (18)$$

$$y = \sum_{i=1}^{n} \alpha_i \left(\mathbf{C}_i\mathbf{x}\right) \qquad (19)$$

By utilizing the paralel distributed compensation principle, a fuzzy controller can be designed for developed fuzzy model. The basic idea in the paralel distributed compensation scheme is to design a linear controller for each one of the local linear model via linear control design techniques and bind them with fuzzy inference algorithm as it has been invoked in fuzzy model development stage. That is;

$$\begin{aligned} &\Gamma\mathbf{1}: \text{IF}\, q_1 \text{ is } FS_1^1 \ \&\, q_2 \text{ is } FS_2^1 \text{ THEN } \mathbf{u} = -\mathbf{K}_1\mathbf{x} + d_1 \\ &\vdots \\ &\Gamma n: \text{IF}\, q_1 \text{ is } FS_1^n \ \&\, q_2 \text{ is } FS_2^n \text{ THEN } \mathbf{u} = -\mathbf{K}_n\mathbf{x} + d_n \end{aligned} \tag{20}$$

$$\mathbf{u} = -\sum_{i=1}^{n} \alpha_i \left(\mathbf{K}_i\mathbf{x} + d_i\right) \tag{21}$$

As a result (21) yields a nonlinear control action which is a nonlinear function of the gap clerance and current values.

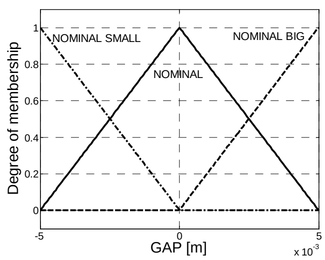

Fig. 2. Membership functions for gap clearance.

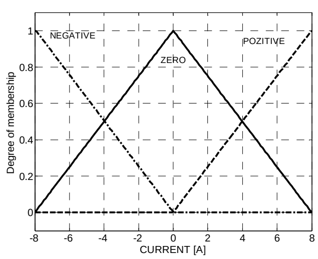

Fig. 3. Membership functions for current.

Practical realization of the outlined fuzzy controller is very much dependent number rules defining the fuzzy model of the system. On the one hand, increase in the number of the rules improves capturing and generalizing poperty fuzzy model and lessens model matching error between actual and fuzzy one. On the other hand, employment of as many as rules means growing calculation complexity and cost and consequently, heavy calculation burden for real time processor. Therefore, firstly, to obtain an admissible fuzzy controller some compromise must be done to select number of rules. Another fundamental issue is the shape of the membership functions. The role of the membership functions is to deduce fuzzy knowledge from the crisp ones. They have convexity features and can be described by well-known bell-shaped or gaussian type distribution functions. Their nonlinearity and as well as smoothes have significant effect on the model maching conditions of fuzzy implications. When the shape gets more complex, their matematical manipulation in real time processor offers heavy calculation cost and fragile practical realization. Therefore, in this research, we choosed 9 rules representing nonlinear dynamics of the system, (2), and determined membership function shapes as triangles and trapezoids as seen in Fig. 2-3.

Subsequently, the structure of the developed fuzzy system takes the form illustrated in Fig. 4.

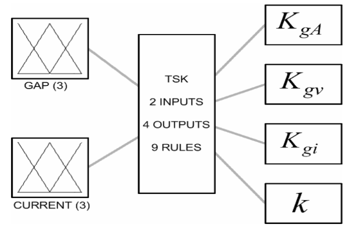

Fig. 4. Fuzzy controller block diagram for magnet A.

3.3 DISTURBANCE OBSERVER DESIGN

Admissible operation of the state space controller relies on the availability of full state measurements. Our system is lack of velocity sensor, thereby, velocity must be recostructed from the measurable state variables. One possibility is to acquire the velocities from gap sensor measurements via numerical derivative techniques. Yet, the noise arising from the numerical derivative calculation is a seious practial difficulty and additionally, employment of low pass filter to reduce the noise level leads to time delay problem.

However, since the system is observable, the velocity value can be reconstructed from a state observer. The classical observer suffers from the outer disturbances and parameter mismatches. Including the outer disturbance value as state variable with a known dynamics can improve the estimation property and at the same time, if the defined disturbance dynamics matches the actual one, acting disturbance value cen be easily observed and utilized for robust control puposes via simply feed-forwarding technique. Here, to construct velocity, zero order disturbance observer design will be derived via extending system matrices by inclusion of disturbance value as state variable;

$$\frac{d}{dt}\hat{\mathbf{x}}_o = \mathbf{A}_o\hat{\mathbf{x}}_o + \mathbf{B}_o\mathbf{u}_o \tag{22}$$

$$y_o = \mathbf{C}_o\hat{\mathbf{x}} \tag{23}$$

$$\begin{bmatrix} \Delta\dot{\hat{z}}_A \\ \Delta\ddot{\hat{z}}_A \\ \dot{\hat{F}}_{dA} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{3K_a}{M} & 0 & \dfrac{3}{M} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{z}_A \\ \Delta\dot{\hat{z}}_A \\ \hat{F}_{dA} \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{3K_b}{M} \\ 0 \end{bmatrix} \Delta i_A \tag{24}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{z}_A & \Delta\dot{\hat{z}}_A & \hat{F}_{dA} \end{bmatrix}^T \tag{25}$$

$$\frac{d}{dt}\hat{\mathbf{x}}_o = \mathbf{A}_o\hat{\mathbf{x}}_o + \mathbf{B}_o\mathbf{u}_o + \mathbf{L}_o\left(\Delta z_A - \mathbf{C}_o\hat{\mathbf{x}}_o\right) \tag{26}$$

Where, the subscript "$_o$" represents extended state space matrices and vectors of the disturbance observer design equations, superscript " ^ " stands for observed state variables and **L** shows observer gain vector. **L** is determined to give faster dynamic response than the controller.

The outlined control algorithms for both linear and fuzzy approaches are depicted in Fig. 2. & 3. Furthermore to obtain generalized axes variables, (*z, α, β*), an axis transformation matrix, *T*, is derived via the utilization of geometricall relationship among gap clearance of hybrid electromagnets.

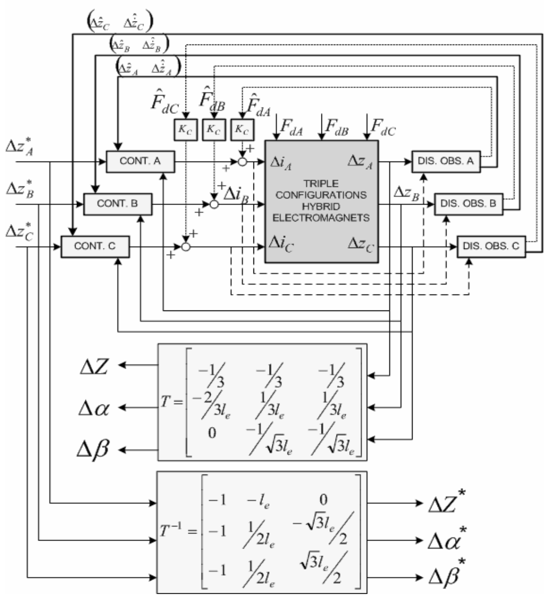

Fig. 5. Linear control system block diagram.

4. EXPERIMENTAL RESULTS

The parameter data set given in Table 1. is utilized experimental studies. Desired poles of the controllers and observers are decided by using Kessler's canonical form for both linear and fuzzy control cases. In Table 1., τ is the time constant of the Kessler's polynomial and γ is the stability index. The former one defines time response speed for the designed controller and in the meanwhile, stability index specifies wave shape of the controller such as oscilatory or less oscilatory for changing reference inputs and acting outer disturbance situations.

Table 1. System Specifications

k	2.01×10^{-5}	$[NM^2/A^2]$	M	9.15	[KG]
I_m	12.4275	[A]	i_0	0	[A]
L_m	0.003	[m]	z_0	0.00734	[mm]
μ_m	1.09		g	9.81	$[kg/(m/sec^2)]$
τ	0.07	[sec]	γ	2.0	
K_{b0}	4.5053	[N/A]	K_{a0}	5398.3	[N/m]

For fuzzy controller design, (2) is linearized over 9 attainable operating points. According to specified operating point, the plant parameters are obtained and controller gains are decided via help of Kessler's polinomial.

For experimental verification purposes, an experimental test bench comprised of triple configuration of hybrid electromagnets is designed as seen in Fig. 1. Controller design issues have been handled by employing Pentium-IV personal computer by using Matlab package. Controller realizations are carried out in digital form via utilization of dSpace 1103, single processor multi-function data acquisition board.

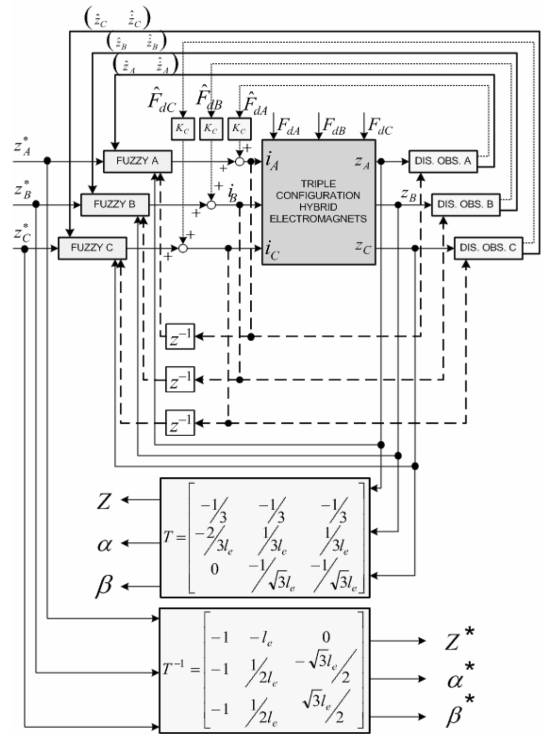

Fig. 6. Fuzzy control systems block diagram.

As a first experimental study, for wide range of gap clerance change, *±3 [mm]*, reference gap clerance tracking response has been investigated without feed-forwarding the estimated disturbance. The experimental results for this experiment are given in Fig. 7-8 for both absolute z and β axes. As seen from the Fig. 7-8, the proposed fuzzy controller design technique shows superiority over the linear counterpart for wide gap clerance change. It verifies that if wide range of the gap clerance change is desired in operations, particularly for high performance control and as well as improved stability conditions, classical linear controller design approaches can not provide required control action by itself with fixed structure. Again, the second experimental test has been conducted for reference tracking performance by feed-forwardeding the estimated disturbance through control path for both linear and fuzzy control cases. Experimental results of this test are given in Fig. 9-10 for absolute z and β axes. As seen from Fig. 9-10, the estimated disturbance feed-forwarding can improve the system performance for both of the experimented control approaches. Moreover, the linear controller design approach needs some more consideration to reduce overshoot and as well as lessen the oscilatory behaviour, while the fuzzy approach keeps on going its supremacy. The last experiment has been about behaviour of the discussed controller structures in the case of outer disturbance excitation. To investigate such a case, *1.2 [kg]* pay-load is applied at *0.874 [sec]* to center of mass of the system for *z axis*, and the results depicted in Fig. 11-12 are obtained. From this results we can conlude that employment of the outlined fuzzy control approach not only improves the system's tracking performance but also considerably improves robustness for external disturbance excitations.

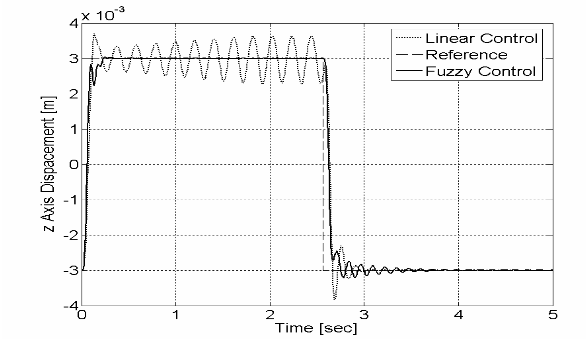

Fig. 7. *z* axis time response for reference change without disturbance feed-forwarding.

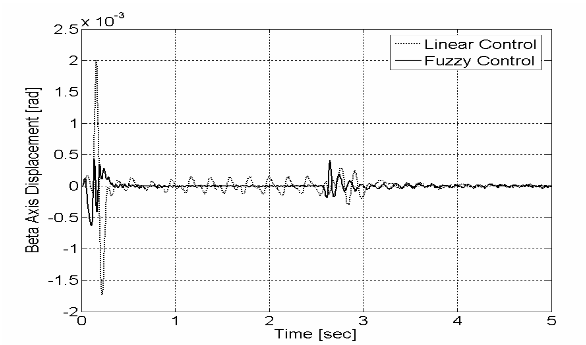

Fig. 8. β axis time response for reference change without disturbance feed-forwarding.

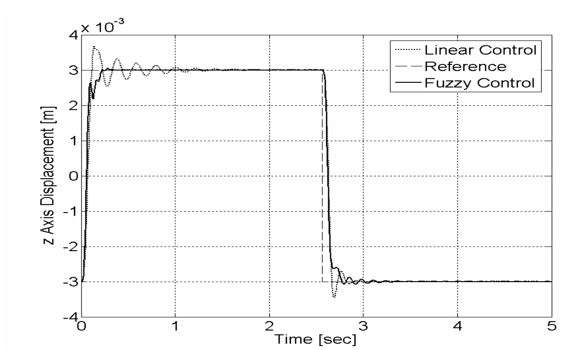

Fig. 9. *z* axis time response for reference change with disturbance feed-forwarding.

5. CONCLUSION & FUTURE PROSPECTS

In this paper, to extend the operating range and improve the robustness of an electromagnetic suspension system, comprised of triple configuration of electromagnets, a TSK fuzzy model-based fuzzy control approach is proposed. Furthermore, to reconstruct the immeasurable gap clearance velocity and to employ the benefit of disturbance estimation, the zero order disturbance observer design issue is described and verification & comparative studies over the linear design counterpart have been demostrated through experimental results. The proposed fuzzy control technique, surely, shows superiority over conventional ones.

As future work, firstly, we are planing to investigate the stability conditions of fuzzy controller via numerical techniques, then, to extend this research through active vibration control area by utilizing the described magnetic suspension system as an actuator in conjunction with mechanical elements.

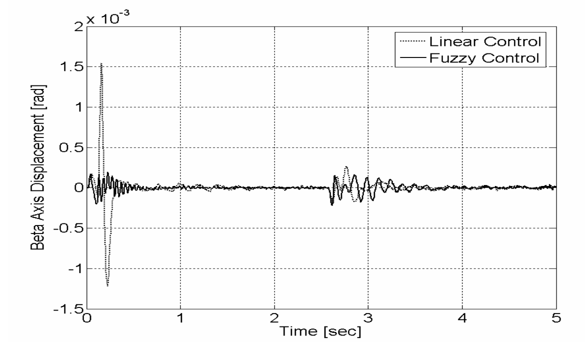

Fig. 10. β axis time response for reference change with disturbance feed-forwarding.

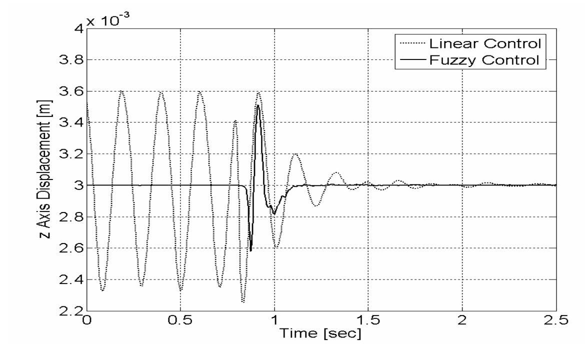

Fig. 11. β axis time response for *1.2 [kg]* pay-load disturbance acting @ *0.874 [sec]* without disturbance feed-forwarding.

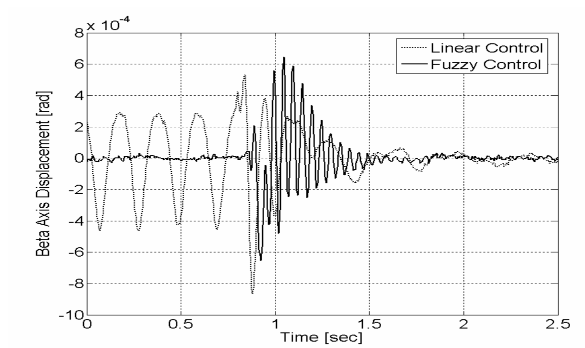

Fig. 12. β axis time response for *1.2 [kg]* pay-load disturbance acting @ *0.874 [sec]* without disturbance feed-forwarding.

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