消費エネルギーを最小化するための区間の速度制約を考慮した列車運転曲線の最適化

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The design of an optimal running curve for train operation considering the sectional-speed constraints with the aim of minimizing the energy consumption

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This paper describes a new approach for creating an optimal running curve of a train considering the sectional-speed constraints with the aim of minimizing the energy consumption, given the basic constraints of running time and limited motive force. A parameterization method of the input motive force profile is proposed for mathematically demonstrating the relationship between the input motive force and the instant time; therefore, the running curve can be obtained by simply solving the dynamic equation of train, given the profile of input motive force. For such approach, the energy consumption and the stated-above constraints are formulated as mathematically functions depending on the parameters of this profile, then a constrained optimization problem is established for minimizing the energy consumption to find the optimal profile of the input motive force. The effectiveness of proposed method is verified by computer simulations.

キーワード：電気鉄道、運転曲線、最適化、パラメータ法、消費エネルギー、断面スピード制約
(Railway, running curve, mathematical optimization, parameterization method, energy consumption, sectional-speed constraint)

1. Introduction

Railway transportation is facing increasing pressure to reduce its energy consumption due to increasing concerns for environment issues. Since the running curve directly affects the total energy consumption, many researchers have considered how to obtain an optimal running curve so that this energy consumption is minimized. Those researches can be grouped into two categories as coasting control and general control [1]. In coasting control, control algorithms are proposed to find appropriate coasting points at which the traction motors are completely switched off. For example, genetic algorithm is implemented to search for these points [2]; and artificial neural network is applied to search for optimal coasting speed to minimize the social cost [3]. Meanwhile, general control seeks for the optimal running curve by searching the train speed at various positions of all three modes of train operation (i.e. powered, coasting, and braking modes). There are numerous algorithms used to obtain these optimal control sequences of train such as Pontryagin's maximum principle [4], Genetic algorithm [5], and Dynamic programming [6]. For those approaches, the optimal running curve is obtained by searching for the speed of train at various positions, so those algorithms are in computational burden due to their generic nature. In [7], the authors proposed an approach to find the optimal running curve based on a novel parameterization method of the running curve. With this approach, since constraints guaranteeing the continuity of train speed profile must be considered, the established optimization problem becomes complicated when many parameterization functions of running curve are used. For example, in case of many sectional-speed constraints considered, using numerous parameterization functions makes the optimization problem very difficult to obtain the solution. To overcome this drawback, in this paper, the authors proposed another approach in applying the parameterization method. Instead of parameterizing the relationship between position and speed of train, parameterizing the relationship between the input motive force and time is used. For this approach, the constraints for continuity conditions are not appeared in the optimization problem, making it easier to solve.

The remains of this paper are outlined as follows: *Section 2* presents dynamic model of train and formulates the energy consumption. *Section 3* proposes a new approach of the parameterization method and establishes the constrained optimization problem. *Section 4* presents simulation results. *Section 5* is about conclusions.

2. Mathematical model of train operation

<2.1> Dynamic equation of the train

The dynamic equation of the train follows Newton's law:

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{M}\left[F_{motor}(v) - F_{mech}(v) - R(x,v)\right] \\ \dfrac{dx}{dt} = v \end{cases} \quad (1)$$

Where, x - position of train; v - speed of Train; M - total mass of train; $R(x, v)$ - total resistances of train.

The input motive force of train $F_{motor}(v)$ can be expressed by the following equations for two cases of train operation:

Case 1: Powered mode: $u_{motor} \geq 0$

$$F_{motor}(v) = \begin{cases} u_{motor} \times F_0, 0 \leq v \leq v_0 \\ u_{motor} \times \frac{F_0 v_0}{v}, v_0 \leq v \leq v_{max} \end{cases} \quad (2)$$

Case 1: Regenerative braking mode: $u_{motor} < 0$

$$F_{motor}(v) = \begin{cases} u_{motor} \times F_1, 0 \leq v \leq v_1 \\ u_{motor} \times \frac{F_1 v_1}{v}, v_1 \leq v \leq v_{max} \end{cases} \quad (3)$$

Where, u_{motor} - the control signal of input motive force, $u_{motor} \in \mathbf{R}; -1 \leq u_{motor} \leq 1$; $F_0 v_0$ - the maximal power in powered mode; $F_1 v_1$ - the maximal power in regenerative braking mode.

The mechanical braking force $F_{mech}(v)$ can be expressed by the following equation in braking mode of train operation:

$$F_{mech}(v) = u_{mech} \times F_{max} \quad (4)$$

Where, u_{mech} - the control signal of input mechanical braking force, $u_{mech} \in \mathbf{R}; 0 \leq u_{mech} \leq 1$; F_{max} - the maximal mechanical braking force.

<2.2> Constraints of the train operation

Total running time:

$$T_{total} = \int_0^T dt \quad (5)$$

T_{total} - total running time is given.

Distance between two stations:

$$x_{(t=0)} = 0; \; x_{(t=T_{total})} = L \quad (6)$$

Where, L - the distance between two stations.

Speed constraints:

$$v_{(x=0)} = 0; \; v_{(x=L)} = 0 \quad (7)$$
$$v_{(0<x<L)} \leq v_{limit}(x) \quad (8)$$

Constraint (8) guarantees: the train starts at a station and stops at next station; constraint (9) guarantees the sectional-speed limit of train between two stations.

Input motive force and mechanical braking force:

From Section (2.1), the inequality constraints are obtained:

$$-1 \leq u_{motor} \leq 1; \; 0 \leq u_{mech} \leq 1 \quad (9)$$

<2.3> Energy consumption of the train

The energy consumption of train can be calculated as follows:

$$E = \int_0^T P_{motor} v dt \quad (10)$$

Where, T - the total running time of train; the supplied power for motor/regenerative power from motor P_{motor} can be expressed by the following equation:

$$P_{motor} = \begin{cases} \frac{1}{\eta} \times F_{motor} \times v, u_{motor} \geq 0 \\ \eta \times F_{motor} \times v, u_{motor} < 0 \end{cases} \quad (11)$$

Where, the efficiency of motor η depends on both the motor force and speed of train as a relationship: $\eta = \eta(F_{motor}, v)$. $\eta(F_{motor}, v)$ shows the characteristics of motor, and in general, it is only determined by experimental methods.

From expressions derived at *Section 2.1*, *Section 2.2*, and *Section 2.3*, the energy consumption depends on F_{motor} and F_{mech}; thus, it also depends on $u_{motor}(t)$ and $u_{mech}(t)$, given the constraints of train operation shown in *Section 2.2*. Therefore, by determining u_{motor} and u_{mech} in the time range of $[0, T]$ to minimize the energy consumption, we can obtain the optimal profiles of the input motive force F_{motor} and the mechanical braking force F_{mech}, and then the optimal running curve. In next section, we will present establishing an optimization problem of the energy consumption based on the parameterization method of control signals ($u_{motor}(t)$ and $u_{mech}(t)$).

3. The constrained optimization problem based on a parameterization method

<3.1> A parameterization method of control signal

In order to find the optimal control signal profiles of $u_{motor}(t)$ and $u_{mech}(t)$ so that the energy consumption is minimal, we assume that these control signals can be expressed by mathematical functions versus time, and these functions will be completely determined through their parameters. For simplicity, the control signals $u_{motor}(t)$ and $u_{mech}(t)$ are demonstrated in the common form as follows:

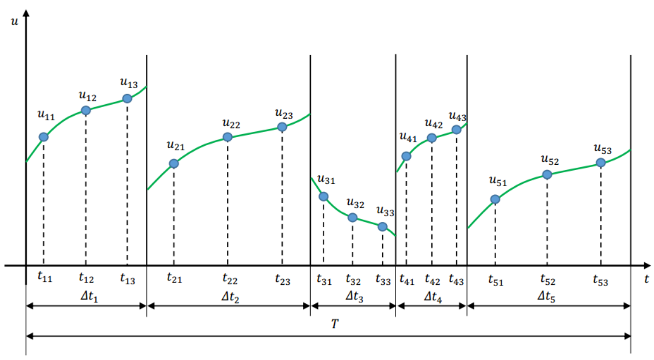

Fig. 1. Parameterization of control signal versus time

From the parameterization method shown in Fig. 1, the control signal consists of N_Δ sections with different length ($\Delta t_i \in \mathbf{R}$; $i = 1,2, \dots, N_\Delta$), and the control signal at each section is parameterized by N_p parameters ($u_{ij}, i = 1,2, \dots, N_\Delta; j = 1,2, \dots, N_p$; $N_\Delta \in \mathbf{N}$; $N_u \in \mathbf{R}$).

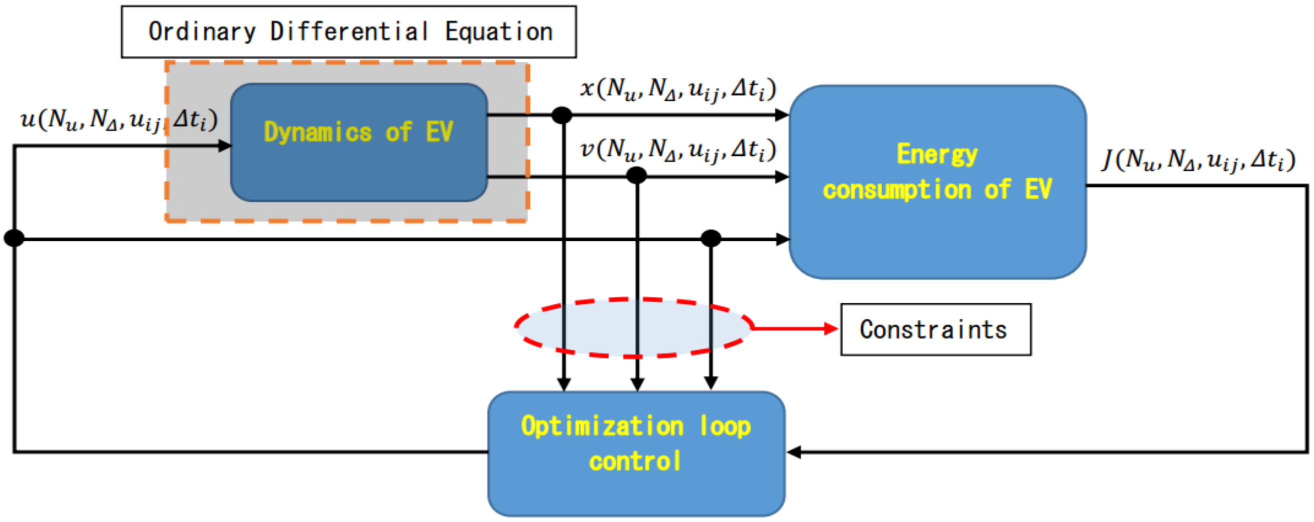

Fig. 2. Optimization algorithm based on the parameterization method of control signal.

It is clearly seen that the control signal consists of piecewise continuous functions which can be mathematically expressed by Lagrange functions as follows:

$$u(t) = \sum_{j=1}^{N_u} \left(u_{ij} \prod_{k=1,k\neq j}^{N_u} \frac{t - t_{ik}}{t_{ij} - t_{ik}} \right); t \in \Delta t_i \qquad (12)$$

At this point, the control signal is parameterized by parameters as follows: $\boldsymbol{P} = (N_\Delta, N_u, \Delta t_i, u_{ij}); i = 1,2, \dots, N_\Delta; j = 1,2, \dots, N_u$. Where, N_Δ, N_u are integer numbers; $\Delta t_i, u_{ij}$ are real numbers.

<3.2> Construction of energy optimization problem based on the parameterization method

A. The energy consumption: Objective function

Dynamic model of the train can be re-written as below:

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{M}\left[F_{motor}(v) - F_{mech}(v) - R(x,v)\right] \\ \dfrac{dx}{dt} = v \\ \dfrac{dE}{dt} = P_{motor} v \end{cases} \qquad (13)$$

Equation (13) can be shorten further as follows:

$$\frac{dX}{dt} = f(X,U); \mathrm{t} \in [0; T_{total}] \qquad (14)$$

Where,

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} v \\ x \\ E \end{bmatrix}; U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_{motor} \\ u_{mech} \end{bmatrix};$$

$$f(X,U) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{M}\left[F_{motor}(v) - F_{mech}(v) - R(x,v)\right] \\ v \\ P_{motor} v \end{bmatrix} \qquad (15)$$

The energy consumption E is obtained by solving the differential equation (12) or (13), given the control signal U, i.e. $E = X_3(T_{total})$. In other words, the energy consumption can be expressed through the parameters of the control signal as follows:

$$E = \mathcal{F}(N_\Delta, N_u, \Delta t_i, u_{ij}) \qquad (16)$$

B. Constraints of the train operation

Total running time:

From equation (5), the following equality is obtained:

$$\sum_{i=1}^{N_\Delta} \Delta t_i = T_{total} \qquad (17)$$

Speed constraints:

From equation (7), we obtain the initial condition and final condition for the differential equation (14).

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; X(T_{total}) = \begin{bmatrix} 0 \\ 0 \\ X_3 \end{bmatrix} \qquad (18)$$

From in-equation (8), the inequality constraints are obtained as follows. By solving the differential equation (14) with the initial equation stated in (17), given the control signal U, we can obtain the trajectory of state $X(t)$ in the time range of $[0; T_{total}]$, then the relationship between position and speed of train is expressed as $v(x) = X_1(X_2)$, so the inequality constraint is obtained:

$$X_1(X_2) \leq \mathrm{v}(X_2) \qquad (19)$$

It should be noted that $X = [X_2(t), X_2(t), X_3(t)]^T$ depends on the parameters of control signal $(N_\Delta, N_u, \Delta t_i, u_{ij})$, so those inequality constraints also depends these parameters.

Input motive force and mechanical braking force:

From in-equations (9), we can obtain the inequality constraints as below:

$$\begin{cases} -1 \le u_{ij} \le 1, & u_{motor} \\ 0 \le u_{ij} \le 1, & u_{mech} \end{cases} \quad (20)$$

From the equality constraints stated in (17), (18) and inequality constraints stated in (19), (20), we can re-write in the short forms as below:

$$h(N_\Delta, N_u, \Delta t_i, u_{ij}) = 0 \quad (21)$$

$$g(N_\Delta, N_u, \Delta t_i, u_{ij}) \le 0 \quad (22)$$

At this point, from (16), (21) and (22), we can obtain the mixed-integer constrained optimization problem as below; the proposed method can be briefly demonstrated in Fig. 2.

$$\begin{cases} E = \mathcal{F}(\boldsymbol{P}) \\ h(\boldsymbol{P}) = 0 \\ g(\boldsymbol{P}) \le 0 \\ \boldsymbol{P} = (N_\Delta, N_u, \Delta t_i, u_{ij}) \\ i = 1,2, \dots, N_\Delta; j = 1,2, \dots, N_u \\ N_\Delta, N_u \in \mathbf{N};\ \Delta t_i, u_{ij} \in \mathbf{R} \end{cases} \quad (23)$$

Solving the mixed-integer constrained optimization problem like (23) is not always obtained because it depends on the complexity of objective function as well as constraints. In this paper, we only investigate a special case that is applicable to train operation optimization.

In case of the number of sections of control signal in the time range $t \in [0; T_{total}]$ and the number of parameters of control signal in each section fixed, i.e. $N_\Delta, N_u = constant$, the optimization problem (23) becomes a general constrained optimization problem where the optimized variables are real numbers. Algorithms for solving this problem have been presented in many researches [7,8,9]. In this paper, we investigate a more special case: when $N_u = 1$, the control signal becomes an array of piecewise constants that are similar to the actions of driver in the train operation, so it is useful for driver guidance.

4. Simulation results

The operation parameters of the train and the characteristics of the input motive force are given as Tab. 1 and Fig. 3.

Tab. 1. Operation parameters of the train

Definition	Symbol	Value
Total mass of train	M [ton]	291.32
Distance	L [m]	2000
Motor efficiency	η	0.8336
Gradient	i [$^0/_{00}$]	-
Maximal train speed	v_{max} [km/h]	90
Limit of Train Acceleration	a_{limit} [$\frac{km/h}{s}$]	-

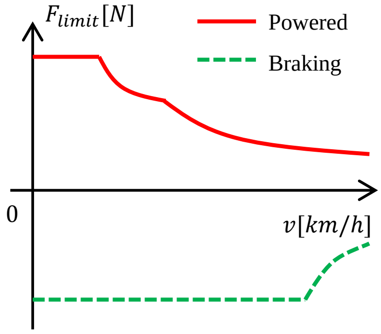

Fig. 3. Characteristics of the input motive force

For verification of the proposed method, in this paper, we only consider the train operation under assumptions: No use of mechanical brake, the constant efficiency of motor $\eta = 0.8336$, and the constant road gradient $i = 2[^0/_{00}]$. Two cases of train operation are investigated i.e. *Case 1*: No sectional-speed constraint; *Case 2*: Sectional-speed constraint.

Case 1: No sectional-speed constraint: ($v_{limit}(x) = v_{max}$)

The running time of train is set as $T_{total} = 135$s. Constraint of the train speed is only $v(x) \le v_{max} (= 90$[km/h]) with any position of the train. Optimization calculations are conducted under different number of parameters of control signal as follows:

$$\boldsymbol{P} = (\Delta t_i, u_{i1}); i = 1,2, \dots, N_\Delta \quad (24)$$

Where, $N_u = 1$; $N_\Delta = 3,4,5$.

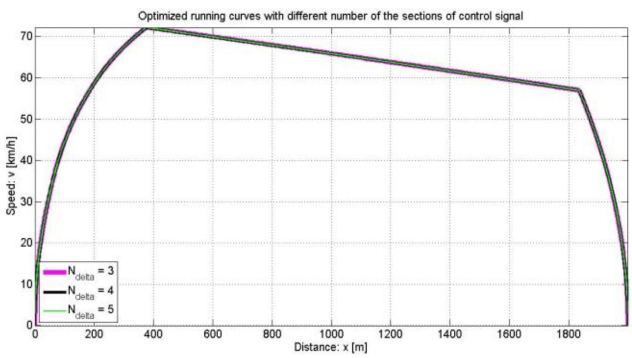

Fig. 3. Optimized running curves of the train with different number of the sections of control signal.

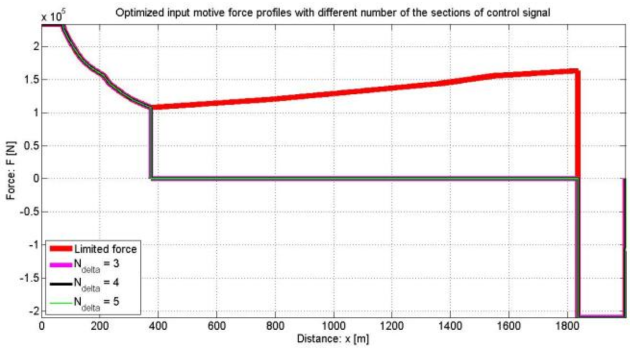

Fig. 4. Optimized input motive force profiles for minimizing the energy consumption of train operation with different number of the sections of control signal.

Simulation results are shown in Fig. 3 and Fig. 4. It should be noted that when no sectional-speed constraint is required, and the running time of train is set long enough (where $T_{total} = 135$s), optimized running curves consist of three sections according to three modes of train operation i.e. powered mode, coasting mode and braking mode as shown in Fig. 3. The input motive force profile also does not significantly change when the number of the sections of control signal increases (provided that $N_\Delta \geq 3$), as shown in Fig. 4. In the three cases of N_Δ ($N_\Delta = 3,4,5$), the minimal energy consumption is same as 13.1015[kWh].

Consider a case where the running time of train is set to be short enough as $T_{total} = 115$ s. Fig.5 and Fig. 6 show optimized running curve and optimized input motive force profile with the number of the sections of control signal chosen as $N_\Delta = 4$.

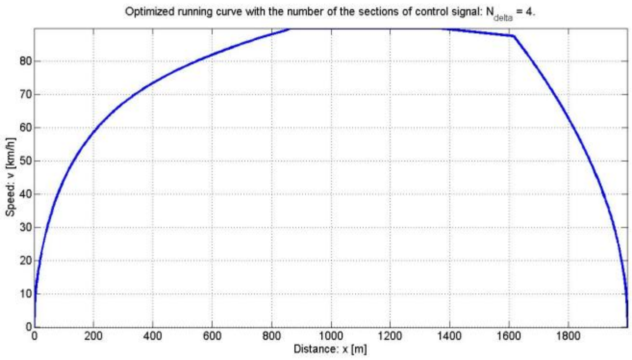

Fig. 5. Optimized running curve of the train with the number of the sections of control signal: $N_\Delta = 4$.

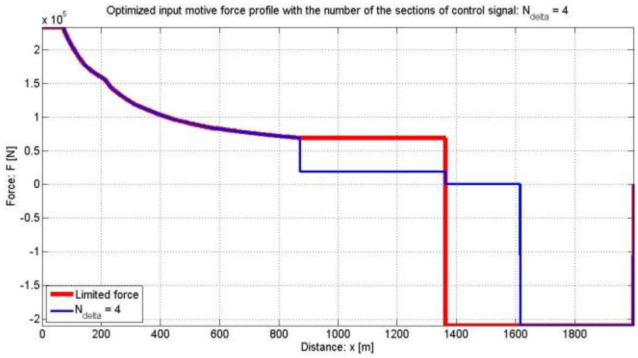

Fig. 6. Optimized input motive force profile for minimizing the energy consumption of train operation: $N_\Delta = 4$.

Fig. 5 shows that due to the constraint of speed limit $v \leq v_{max} = 90$[km/h], the train is accelerated from zero until its speed reaches 90[km/h], then controlled to keep its speed at a constant value in coasting mode. After that, it is decelerated until destination. Further, Fig. 6 shows that due to constraint of the running time, in one part of coasting mode, the train is still powered with an appropriate motive force to keep running time of the train as desired. The minimal energy consumption E in this case is 19.2926[kWh].

Case 2: Sectional-speed constraint: ($v_{limit}(x) \leq v_{max}$)

The running time of train is set as $T_{total} = 135$s. The train speed must be lower than $v_{max}(= 90$[km/h]) with any position of train. Besides, the sectional-speed constraint is set as $v(x) \leq 60$[km/h] when $800 \leq x \leq 1200$ [m]. Optimization calculations are conducted under different number of the sections of control signal as follows:

$$\boldsymbol{P} = (\Delta t_i, u_{i1}); i = 1,2, \dots, N_\Delta \quad (25)$$

Where, $N_u = 1$; $N_\Delta = 6,7,8$.

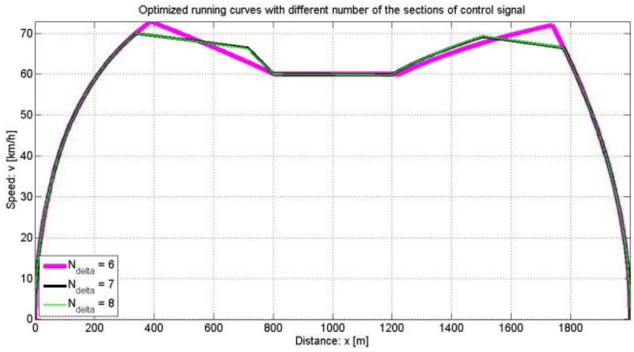

Fig. 7. Optimized running curves of the train with different number of sections of control signal.

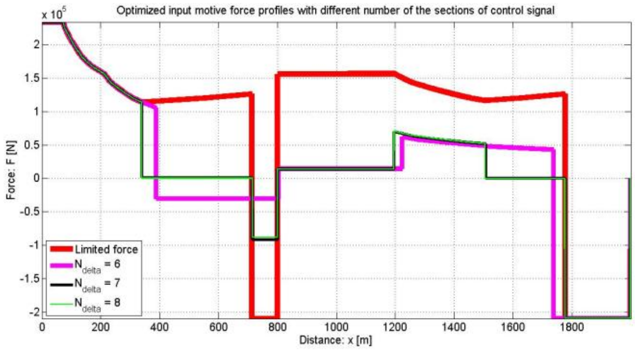

Fig. 8. Optimized input motive force profiles for minimizing the energy consumption of train operation with different number of sections of control signal.

Simulation results are shown in Fig. 7 and Fig. 8. It should be noted that when position of the train is in [800; 1200][m], its speed is controlled to be lower than 90[km/h]. When the number of the sections of control signal increases, the optimized running curves converge to one running curve, so-called the optimal running curve, as shown in Fig. 7. The minimized energy consumption E is respectively 16.4874[kWh]; 15.1518[kWh]; 15.1371[kWh] according to differential number of the sections: $N_\Delta = 6,7,8$. It clearly seen that $N_\Delta = 7$ should be chosen. Fig. 8 shows that the optimized profiles of input motive force in each case of N_Δ are within the limited force profile given in Fig. 3. Determining N_Δ depends on the number of sectional-speed constraints given and desired accuracy of the obtained solution.

5. Conclusions & Future Researches

The research proposed a methodology to find input motive force profile and then obtain the optimal running curve of the train with the aim at minimizing the energy consumption.

It is crucial to select the number of sections of control signal. This paper presented the simulation results with different number of these sections, then an appropriate number is voted. Future research will investigate a method to determine the optimal number of sections of control signal.

It should be noted that the proposed method requires solving the ordinary differential equation (ODE) of train dynamic model in each iterative loop of the optimization process which costs much time for integration calculation if the running time is long enough; therefore, fast algorithms for solving this ODE are very important. There are many available integrator packages of ODE such as: Sundials [10] or DAESOL [11]. It will be useful if the proposed method is used together with these available software packages to improve performance of the optimization algorithm.

文　　献

(1) Shaofeng Lu, "Optimizing Power Management Strategies for Railway Traction Systems", *Doctoral thesis*, *The University of Birmingham*, UK, 2011.

(2) C. Chang, S. Sim, "Optimizing Train Movements through Coast Control Using Genetic Algorithms", *IEE Proceeding – Electric Power Applications*, Vol.144, No.1, pp.65-73, 1997.

(3) H. -J. Chuang, C. -S. Chen, C. -H. Lin, C. -H. Hsieh, C. -Y. Ho, "Design of optimal coasting speed for saving social cost in mass rapid transit systems", *Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, pp.2833-2839, 2008.

(4) R. Liu, L. M. Golovitcher, "Energy-efficient operation of rail vehicles", *Trans-portation Research Part A: Policy and Practice*, Vol.37, No.10, pp.917-932, 2003.

(5) W. Liu, Q. Li, B. Tang, "Energy Saving Train Control for Urban Railway Train with Multi-Population Genetic Algorithm", *International Forum on Information Technology and Applications, IEEE, IEEE Computer Society*, pp.58-63, 2009.

(6) H. Ko, T. Koseki, M. Miyatake, "Application of Dynamic Programming to Optimization of Running Profile of a Train", *Computer in Railway IX*, Vol.15, The Wessex Institute, pp.103-112, 2004.

(7) V.D. Doan, S. Watanabe, T. Koseki, "A Design of Optimal Running Curve for Train Operation Based on a Novel Parameterization Method for Saving Energy", *Computer in Railway XIV*, Vol. 135, pp. 174-190, June 2014.

(8) Hailay Weldegiorgis Berhe, "Penalty function methods using matrix laboratory (MATLAB)", *African Journal of Mathematics and Computer Science Research*, Vol.5(13), pp.209-246, Nov. 2012.

(9) Masafumi Miyatake, Hideyoshi Ko, "Optimization of Train Speed Profile for Minimum Energy Consumption", *IEEJ TRANSACTIONS ON ELECTRICAL AND ELECTRONIC ENGINEERING*, IEEJ Trans 2010, Vol. 5, pp. 263–269.

(10) "Sundials—SUite of Nonlinear and Differential/ALgebraic Equation Solvers (web page and software)", 2009.
Available from: https://computation.llnl.gov/casc/sundials

(11) Bauer, I., Bock, H., Körkel, S., Schlöder, J., "Numerical methods for initial value problems and derivative generation for DAE models with application to optimum experimental design of chemical processes", *Scientific Computing in Chemical Engineering II*, pp. 282–289. Springer, Berlin (1999).