Design Method of Transmitter Voltage and Load Impedance for Multiple Transmitter/Receiver Wireless Power Transfer via Magnetic Resonant Coupling

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Abstract In this paper, we deal with Multiple Transmitter/Receiver Wireless Power Transfer (MTMR-WPT). Multiple transmitters enable longer longitude distance and more efficient WPT by changing transmitter softly to follow receiver motion. Multiple receivers enable more robustness by redundancy of receiver. We propose a formula to transform from MTMR-WPT to single transmitter/receiver WPT and a method to design transmitter side voltage rate and load impedance to realize maximum efficiency and optimal operational condition of MTMR-WPT. Results of numerical calculation and plan of experiments for verification the optimal operational design are expressed.

Keyword Wireless Power Transfer (WPT), Magnetic Resonant Coupling (MRC), Multiple Transmitter, Multiple Receiver, Efficiency Maximizing

1. Introduction

Wireless Power Transfer (WPT) is expected to be as a method for power feeding where and when wiring is difficult. WPT via Magnetic Resonant Coupling (MRC), which was proposed by MIT [1], enables several 10 cm and kW-scale WPT. The transfer distance is longer than the conventional induction type WPT, which can transmit at longest 20 cm. Furthermore, WPT via MRC is robust for misplacement and changing transfer distance [2]. This characteristics has been applied for stationary charging for electric vehicles (EVs) [3], electric railway [4], and dynamic charging for EV [2].

Especially, in the case of long longitudinal WPT, for example, dynamic charging, high frequency current which is originally prepared for power feeding should be applied only in restricted area where WPT is needed from the viewpoint of realizing higher efficiency and Electro-Magnetic Compatibility (EMC). However, if long single transmitter coil is used, high frequency current is applied in some area which is not used for WPT. Therefore, for dynamic charging, transmitter coil should be divided as multiple small transmitter coils. On the other hand, multiple receiver coils is applied for the case of collective charging for multiple loads in an enclosed area or charging for long longitudinal shaped load, for example, electric bus or electric railway.

Some former researches of WPT using multiple transmitters and multiple receivers (MTMR-WPT) has been investigated. For example, there has been investigation of influence caused by coupling between transmitters or between receivers [5], investigation of realization of wider transfer area by using multiple transmitters [6], investigation of load resistance control for multiple receiver WPT [7], and investigation of using single transmitter, repeater and multiple load coils for charging multiple loads [8]. However, proposal of a transform formula from MTMR-WPT system to single receiver and single transmitter (STSR) WPT system, and straight forward form of requirement for maximum efficiency has not been investigated yet, although many researches about STSR-WPT can be easily applied to MTMR-WPT.

In this paper, we propose a transform formula from MTMR-WPT to STSR-WPT, and straight forward form of requirement for maximum efficiency, and verify some cases of it by numerical calculation and experiment.

2. Single transmitter single receiver WPT

2.1. Equivalent Circuit and Efficiency Formula

Circuit of WPT via MRC is shown as Fig. 1. Definition of symbols is available in Tab. 1.

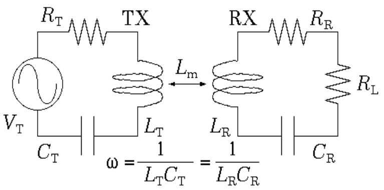


Fig. 1 Circuit diagram of WPT via MRC.

Tab. 1 Definition of Symbols

Symbol	Definition	Value	Unit
V_T	Transmitter (TX) voltage		V
V_R	Receiver (RX) Voltage		V
I_T	TX Current		A
I_R	RX Current		A
R_T	TX Internal Resistance	1	Ω
R_R	RX Internal Resistance	1	Ω
f	Power source frequency	85	kHz
ω	Power source angular frequency	$2\pi f$	rad/s
L_T	TX self-inductance		μH
L_R	RX self-inductance		μH
C_T	TX resonant capacitance	$\frac{1}{\omega^2 L_T}$	(pF)
C_R	RX resonant capacitance	$\frac{1}{\omega^2 L_R}$	(pF)
L_m	Mutual Inductance between TX and RX		μH
R_L	Load resistance		Ω

From circuit calculation, efficiency η, received power P_R is expressed as eq. (1), (2) [2].

$$\eta = \frac{V_R I_R^*}{V_T I_T^*} = \frac{R_L}{R_L + R_R} \cdot \frac{\omega^2 {L_m}^2}{\omega^2 {L_m}^2 + R_T(R_R + R_L)} \quad (1)$$

$$P_R = V_R I_R^* = \frac{R_L \omega^2 {L_m}^2}{\left(\omega^2 {L_m}^2 + R_T(R_R + R_L)\right)^2} V_T \quad (2)$$

2.2. Requirement for Maximum Efficiency or Higher Power

According to eq. (1), η increases when L_m increases or R_L approaches a certain value. Efficiency becomes maximum when R_L is $R_{L,\eta\max}$ as eq. (3).

$$R_{L,\eta\max} = \sqrt{\frac{R_R}{R_T}\omega^2 {L_m}^2 + {R_R}^2} \quad (3)$$

Relationship between η and P_R depends on R_L and L_m. The locus of η and P_R by sweeping R_L or L_m is shown on Fig. 2. From this figure, if higher efficiency is needed, fixing R_L as $R_{L,\eta\max}$ or higher L_m is needed. If higher power is required, higher V_T or higher R_L than $R_{L,\eta\max}$ is needed although the latter solution is less efficient.

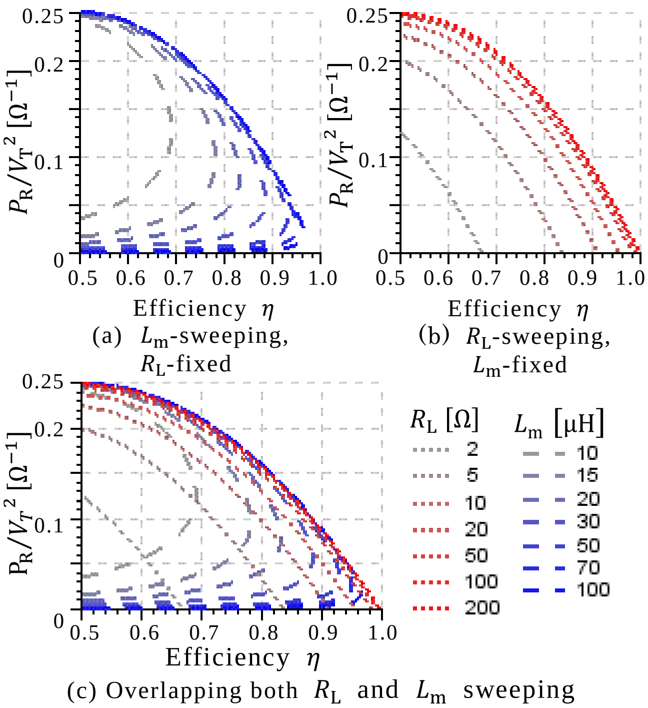


Fig. 2 $\eta - P_R$ (fixed V_T) map

3. General discussion on maximizing efficiency of MTMR-WPT

3.1. Qualitative Discussion on MTMR-WPT

If there are multiple transmitters, more power should flow from a transmitter which has higher L_m from the viewpoint of efficiency.

However, in MRC-WPT, impedance measured from transmitter (V_T/I_T) becomes higher when L_m is higher. It means, higher L_m suppress transmitter current, and lower L_m increases transmitter current. From this characteristic, more power flows from a transmitter which has lower L_m. Therefore, voltage of transmitters should not be the same and should be controlled for achieving higher efficiency.

If there are multiple receivers, power flow can be controlled by R_L, so efficiency can be changed by R_L variation. Therefore, we investigate condition for maximum efficiency of V_T rate and R_L values for MTMR-WPT.

3.2. Transform formula from MTMR-WPT to Single-Transmitter/Receiver WPT

If mutual inductance between transmitters or between receivers, such as, TX1-TX2, RX1-RX2, TX k-TX k', RX k-RX-k' and so on, can be ignored, circuit of N-TX and M-RX WPT via MRC is shown in Fig. 3 and circuit equations are expressed as eq. (4)—(8).

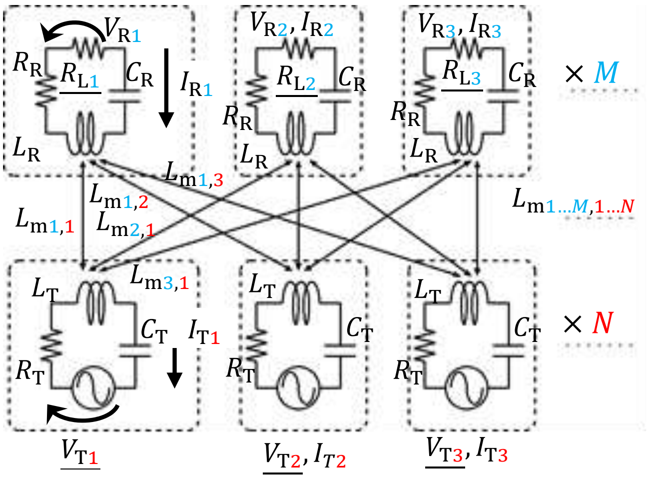


Fig. 3 Circuit diagram of MTMR-WPT

When transform formula of equivalent load resistance $\widetilde{R_{\mathrm{L}}}$ and equivalent squared mutual inductance $\widetilde{L_{\mathrm{m}}}^2$ expressed as eq. (9)—(10) is applied, formula of efficiency becomes eq. (12). It has the same form of STSR system expressed in eq. (1).

$$\boldsymbol{V}_{\mathrm{T}} = \begin{pmatrix} V_{\mathrm{T1}} \\ \vdots \\ V_{\mathrm{T}N} \end{pmatrix} = R_{\mathrm{T}}\boldsymbol{I}_{\mathrm{T}} - j\omega \boldsymbol{L}_{\mathrm{m}}{}^{T}\boldsymbol{I}_{\mathrm{R}} \tag{4}$$

$$\boldsymbol{V}_{\mathrm{R}} = \begin{pmatrix} V_{\mathrm{R1}} \\ \vdots \\ V_{\mathrm{R}M} \end{pmatrix} = j\omega \boldsymbol{L}_{\mathrm{m}}\boldsymbol{I}_{\mathrm{T}} - R_{\mathrm{R}}\boldsymbol{I}_{\mathrm{R}} \tag{5}$$

$$\boldsymbol{V}_{\mathrm{R}} = \mathrm{diag}(R_{\mathrm{L1}}, R_{\mathrm{L2}}, \cdots, R_{\mathrm{L}M})\boldsymbol{I}_{\mathrm{R}} \tag{6}$$

$$\boldsymbol{L}_{\mathrm{m}} = \begin{pmatrix} L_{\mathrm{m}1,1} & \cdots & L_{\mathrm{m}1,N} \\ \vdots & \ddots & \vdots \\ L_{\mathrm{m}M,1} & \cdots & L_{\mathrm{m}M,N} \end{pmatrix} \quad (M \times N) \tag{7}$$

$$\boldsymbol{I}_{\mathrm{T}} = \begin{pmatrix} I_{\mathrm{T1}} \\ \vdots \\ I_{\mathrm{T}N} \end{pmatrix}, \boldsymbol{I}_{\mathrm{R}} = \begin{pmatrix} I_{\mathrm{R1}} \\ \vdots \\ I_{\mathrm{R}M} \end{pmatrix} \tag{8}$$

$$\widetilde{R_{\mathrm{L}}} = \frac{\boldsymbol{c}^{T}\mathrm{diag}\left(\frac{R_{\mathrm{L1}}}{(R_{\mathrm{L1}}+R_{\mathrm{R}})^2}, \cdots, \frac{R_{\mathrm{L}M}}{(R_{\mathrm{L}M}+R_{\mathrm{R}})^2}\right)\boldsymbol{c}^{*}}{\boldsymbol{c}^{T}\mathrm{diag}\left(\frac{1}{(R_{\mathrm{L1}}+R_{\mathrm{R}})^2}, \cdots, \frac{1}{(R_{\mathrm{L}M}+R_{\mathrm{R}})^2}\right)\boldsymbol{c}^{*}} \tag{9}$$

$$\widetilde{L_{\mathrm{m}}}^2 = \frac{\left(\boldsymbol{c}^{T}\mathrm{diag}\left(\frac{1}{R_{\mathrm{L1}}+R_{\mathrm{R}}}, \cdots, \frac{1}{R_{\mathrm{L}M}+R_{\mathrm{R}}}\right)\boldsymbol{c}^{*}\right)^2}{\boldsymbol{c}^{T}\mathrm{diag}\left(\frac{1}{(R_{\mathrm{L1}}+R_{\mathrm{R}})^2}, \cdots, \frac{1}{(R_{\mathrm{L}M}+R_{\mathrm{R}})^2}\right)\boldsymbol{c}^{*}} \tag{10}$$

$$\boldsymbol{c} = \frac{1}{|\boldsymbol{I}_{\mathrm{T}}|}\boldsymbol{L}_{\mathrm{m}}\boldsymbol{I}_{\mathrm{T}} (M \times 1) \tag{11}$$

$$\eta = \frac{\boldsymbol{I}_{\mathrm{R}}{}^{*}\boldsymbol{V}_{\mathrm{R}}}{\boldsymbol{I}_{\mathrm{T}}{}^{*}\boldsymbol{V}_{\mathrm{T}}} = \frac{\widetilde{R_{\mathrm{L}}}}{\widetilde{R_{\mathrm{L}}} + R_{\mathrm{R}}} \cdot \frac{\omega^2 \widetilde{L_{\mathrm{m}}}^2}{\omega^2 \widetilde{L_{\mathrm{m}}}^2 + R_{\mathrm{T}}(R_{\mathrm{R}} + \widetilde{R_{\mathrm{L}}})} \tag{12}$$

3.3. Requirement for Maximizing Efficiency by Maximizing Equivalent Squared Mutual Inductance Value

In STSR-WPT, higher L_{m} and a certain value of R_{L} gives higher efficiency. However, in MTMR-WPT, both $\widetilde{R_{\mathrm{L}}}$ and $\widetilde{L_{\mathrm{m}}}^2$ depends on not only original mutual inductance matrix $\boldsymbol{L}_{\mathrm{m}}$, but also TX voltage or current rate. Also, load impedance $R_{\mathrm{L1}}, \cdots R_{\mathrm{LM}}$ influences both $\widetilde{R_{\mathrm{L}}}$ and $\widetilde{L_{\mathrm{m}}}^2$. Therefore, requirement of setting $R_{\mathrm{L1}}, \cdots R_{LM}$ and TX current/voltage rate for maximum $\widetilde{L_{\mathrm{m}}}^2$ is needed.

Firstly, requirement of $R_{\mathrm{L1}}, \cdots R_{\mathrm{LM}}$ for maximizing $\widetilde{L_{\mathrm{m}}}^2$ is concluded as eq. (13) by partial differential of $R_{\mathrm{L1}}, \cdots R_{\mathrm{LM}}$. This equation means the same load resistance is needed for maximum efficiency. If this load resistance is set for R_{L0}, $\widetilde{L_{\mathrm{m}}}^2$ and $\widetilde{R_{\mathrm{L}}}$ becomes as eq. (14)—(15).

$$R_{\mathrm{L1}} = R_{L2} = \cdots = R_{LN} \ (= R_{L0}) \tag{13}$$

$$\widetilde{L_{\mathrm{m}}}^2 = \frac{1}{|\boldsymbol{I}_{\mathrm{T}}|^2}\boldsymbol{I}_{\mathrm{T}}{}^{T}\boldsymbol{L}_{\mathrm{m}}{}^{T}\boldsymbol{L}_{\mathrm{m}}\boldsymbol{I}_{\mathrm{T}}{}^{*} \tag{14}$$

$$\widetilde{R_L} = R_{\mathrm{L0}} \tag{15}$$

Secondly, requirement of TX current rate for maximizing $\widetilde{L_{\mathrm{m}}}^2$ is concluded as eq. (16) by using $\boldsymbol{v}_0$ expressed as eq. (17)—(18). This means, current rate should be parallel to a certain vector which has the highest eigenvalue in the eigenvectors of $(N \times N)$ matrix $\boldsymbol{L}_{\mathrm{m}}{}^{T}\boldsymbol{L}_{\mathrm{m}}$.

Thirdly, requirement of R_{L0} is conducted in the same method of STSR-WPT and expressed as eq. (19).

$$\frac{1}{|\boldsymbol{I}_{\mathrm{T}}|}\boldsymbol{I}_{\mathrm{T}} = \frac{1}{|\boldsymbol{v}_0|}\boldsymbol{v}_0 \tag{16}$$

$$\boldsymbol{L}_{\mathrm{m}}{}^{T}\boldsymbol{L}_{\mathrm{m}}\boldsymbol{v}_k = l_k{}^2\boldsymbol{v}_k, l_k{}^2\text{: scalar} \tag{17}$$

$$\boldsymbol{L}_{\mathrm{m}}{}^{T}\boldsymbol{L}_{\mathrm{m}}\boldsymbol{v}_0 = l_0{}^2\boldsymbol{v}_k;\ l_0{}^2 = \max(l_k) \tag{18}$$

$$R_{\mathrm{L0}} = \sqrt{\frac{R_{\mathrm{R}}}{R_{\mathrm{T}}}\omega^2\widetilde{L_{\mathrm{m}}}^2 + R_R{}^2} \tag{19}$$

4. DISCUSSION ON SIMPLIFIED MODEL OF MTMR-WPT

4.1. Case Study of Single Transmitter or Single Receiver Model

We proposes two simple cases to simplify discussion: (a) NTX-1RX system and (b) 1TX-NRX system, as shown in Fig. 4.

(a) NTX-1RX system can be applied in the case of dynamic charging. Dividing TX into multiple coils has merits in efficiency and EMC because it prevents high frequency current in TX when RX is not located nearby. In this system, requirement for maximum efficiency is expressed as eq. (20)—(21). It means, TX current and voltage rate should be set as mutual inductance rate, and R_{T} should be set using total of squared mutual inductance. Also, TX current rate and voltage rate is the same in maximum efficiency mode. It means, if TX voltage rate is controlled to match current rate and voltage rate, efficiency becomes maximum value.

$$\frac{1}{|\boldsymbol{V}_\mathrm{T}|}\boldsymbol{V}_\mathrm{T} = \frac{1}{|\boldsymbol{I}_\mathrm{T}|}\boldsymbol{I}_\mathrm{T} = \frac{1}{|\boldsymbol{L}_\mathrm{m}|}\boldsymbol{L}_\mathrm{m} \quad (20)$$

$$R_\mathrm{L} = \sqrt{\frac{R_\mathrm{R}}{R_\mathrm{T}}\omega^2 \sum_{k=1}^{N} {L_{\mathrm{m}\,1,k}}^2 + {R_R}^2} \quad (21)$$

(b) 1TX-*N*RX system can be seen in the case of collective charging for multiple loads in a sealed area, such as a factory. Requirement for maximum efficiency is expressed as eq. (22). It means every load resistance should have a certain value for maximizing efficiency.

$$R_\mathrm{L1} = \cdots = R_{\mathrm{L}N} = \sqrt{\frac{R_\mathrm{R}}{R_\mathrm{T}}\omega^2 \sum_{k=1}^{N} {L_{\mathrm{m}\,k,1}}^2 + {R_\mathrm{R}}^2} \quad (22)$$

$$P_{\mathrm{R}n} = \frac{\omega^2 {L_{\mathrm{m}\,n,1}}^2}{\omega^2 \sum {L_{\mathrm{m}\,k,1}}^2 + R_\mathrm{T}(R_R + R_\mathrm{L})}{V_\mathrm{T}}^2 \quad (23)$$

Also, the power each receiver gains is expressed as eq. (23). Therefore, if WPT for multiple load collectively especially where mutual inductances can be treated as stable, designing coils to set mutual inductance in proportion to root needed power and after rectifying, use DC-DC converters to change voltages to suitable for the load in order to set load resistances for the same value equivalently.

4.2. Detail of numerical calculation

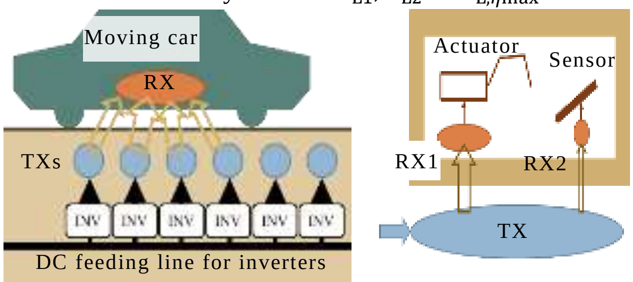
However, requirement for maximum efficiency had been proposed, sensitivity in changing TX voltage ratio and RX load resistance value is not cleared. Therefore, numerical calculation for verify this sensitivity is needed. We verified the case of 2TX-1RX model and 1TX-2RX model in this paper. The values of internal resistance and TX voltage are shown in Tab. 2, and use variables for each load resistance $R_\mathrm{L1}, R_\mathrm{L2}$ and voltage ratio. Mutual inductance is shown in Tab. 3, which is based on measured value in the condition in Fig. 5.

4.3. Calculation result and discussion of 2TX-1RX system

Calculation result of 2TX-1RX system is shown in Fig. 6. This result indicates voltage rate is dominant. Also, received power is shown in Fig. 7, where V_{T2}/V_{T1} is fixed as mutual inductance rate. According to these result, power becomes higher and robustness in errors of voltage rate is enhanced when load resistance is increased. On the other hand, efficiency becomes maximum when load resistance takes $R_{\mathrm{L},\eta\mathrm{max}}$ as expressed as eq. (21). Therefore, load resistance should be set a little higher than $R_{\mathrm{L},\eta\mathrm{max}}$ for gaining both power and robustness in errors of voltage rate. In this case, load resistance should be around 100 Ω, which is higher than $R_{\mathrm{L},\eta\mathrm{max}}$ (≈ 60 Ω).

4.4. Calculation result and discussion of 1TX-2RX system

Calculation result of 1TX-2RX system is shown in Fig. 8. In all cases of Fig. 8, efficiency became maximum where $R_\mathrm{L1} = R_\mathrm{L2} = R_{\mathrm{L},\eta\mathrm{max}}$. Also, value of received power when $V_\mathrm{T} = 1\,\mathrm{V}$ is shown in Fig. 9. According to these results, received power is divided by depending on mutual inductance value. Also, higher power is gained at the expense of efficiency from maximum efficiency where $R_\mathrm{L1}, R_\mathrm{L2} > R_{\mathrm{L},\eta\mathrm{max}}$.

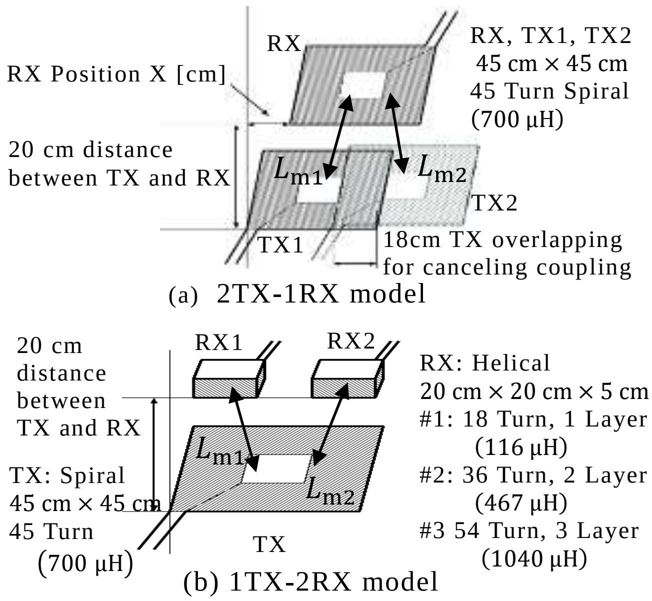


(a) *N*TX-1RX (b) 1TX-*N*RX

Fig. 4. Concept of *N*TX-1RX/1TX-*N*RX system.

Tab. 2. Internal resistance and TX voltage value

Symbol	Value	Unit
R_T	1	Ω
R_R	1	Ω
V_T1	1	V
V_T2	0.1—10	V



(a) 2TX-1RX model

(b) 1TX-2RX model

Fig. 5. Condition for measuring mutual inductance.

Tab. 3. Mutual inductance value.

Case	Condition	L_{m1} [μH]	L_{m2} [μH]	$R_{L,\eta max}$ [Ω]
2TX-1RX	X=22.5 cm; RX on the middle point of 2 TXs	75	75	57
	X=49.5 cm; RX on the TX2	20	120	65
	X=36 cm; Middle point of other 2 cases	48	98	58
1TX-2RX	Use RX #1, #2	17	30	18
	Use RX #2, #3	30	50	28
	Use RX #1, #3	17	50	31

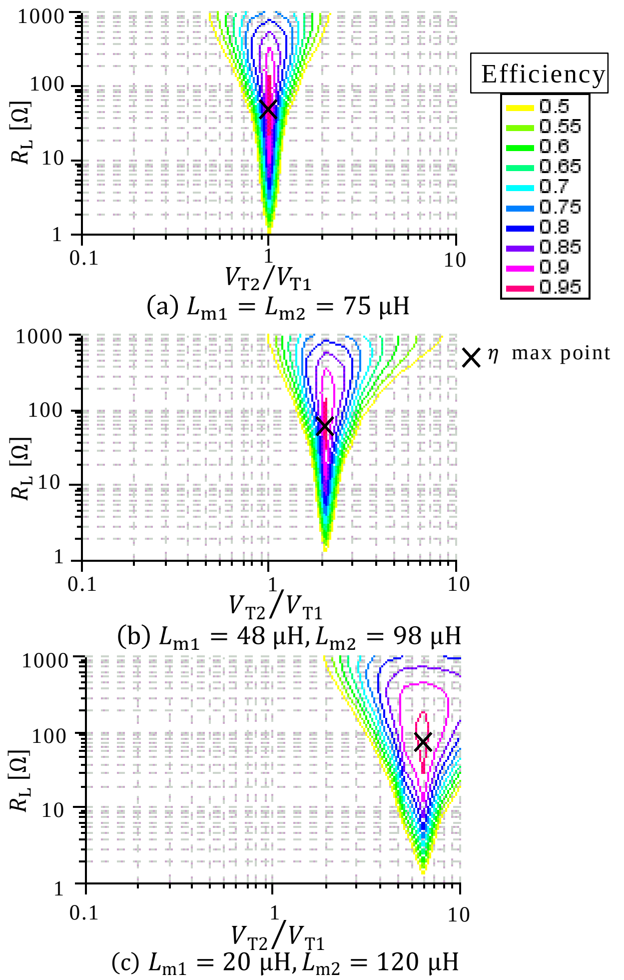


Fig. 6 Numerical calculation result of 2TX-1TX system (Efficiency.)

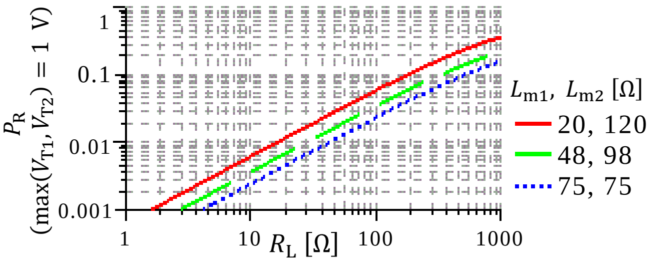


Fig. 7 Numerical calculation result of 2TX-1RX system at $V_{T2}/V_{T1} = L_{m2}/L_{m1}$ (Received Power.)

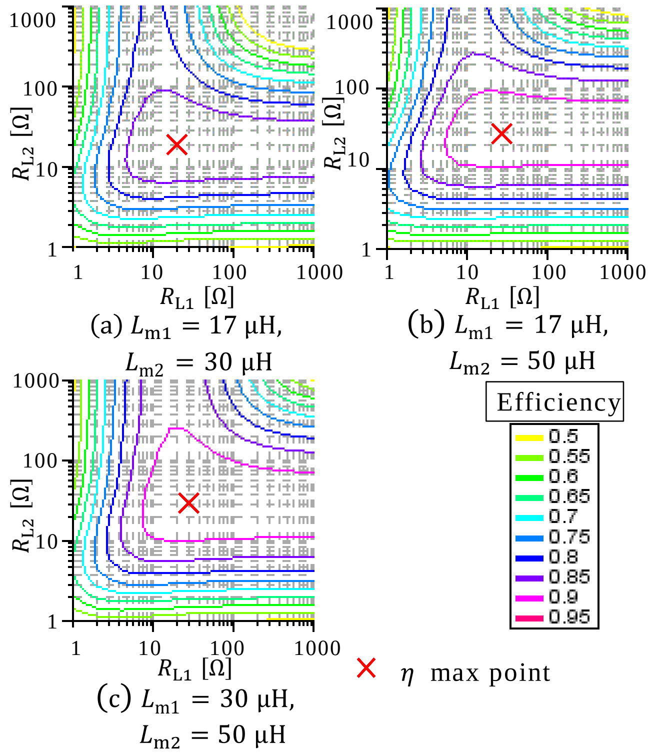


Fig. 8 Numerical calculation result of 1TX-2RX system (Efficiency.)

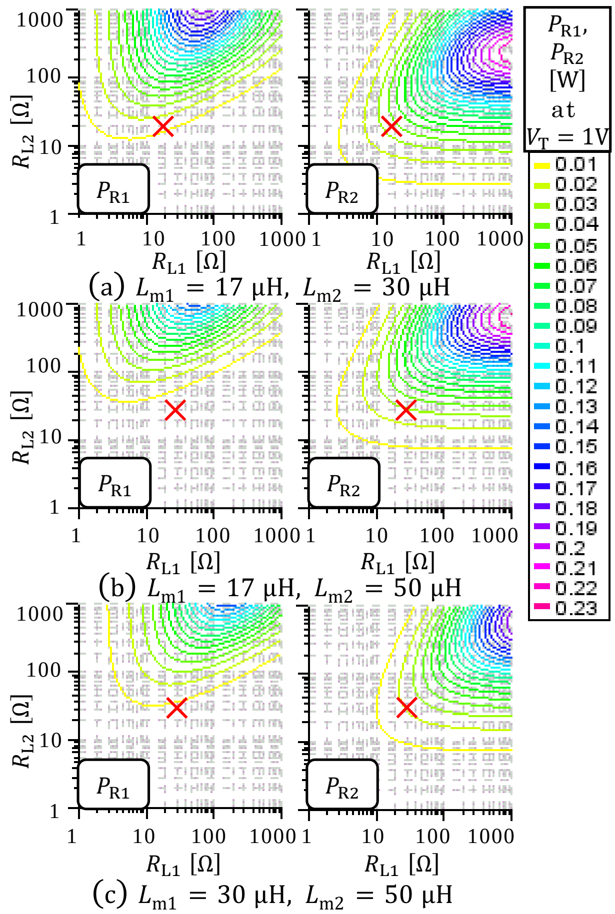


Fig. 9 Numerical calculation result of 1TX-2RX system (Received Power.)

5. EXPERIMENTAL VERIFICATION

5.1. Experimental condition

Experimental condition for 2TX-1RX is shown in Fig. 10, and coil layout is the same as Fig. 5(a). In this time, L_{m1} was 82 μH, and L_{m2} is 87 μH. RX is moved for changing mutual inductances. Several patterns for TXs voltage (phase-synchronized) were applied. Efficiency is calculated from eq.(24), (25) using root mean squared (RMS) voltage of both side of TXs, RMS voltage of RX, shunt resistor R_{s} and load impedance. At the same time, $P_{\mathrm{T1}}/P_{\mathrm{R}}$, $P_{\mathrm{T2}}/P_{\mathrm{R}}$, which means required value of each TX transmitting power when P_{R} is 1 W, and $P_{\mathrm{R}}/\max\left(V_{\mathrm{T1}}{}^2, V_{\mathrm{T2}}{}^2\right)$, which means receivable power when maximum TX voltage is 1V, are calculated.

$$\eta = \frac{\mathrm{rms}(V_{\mathrm{R}})^2 \cdot 2R_{\mathrm{s}}/R_{\mathrm{L}}}{\sum \mathrm{rms}(V_{\mathrm{T}k+})^2 - \mathrm{rms}(V_{\mathrm{T}k\Delta})^2 - \mathrm{rms}(V_{\mathrm{T}k-})^2} \quad (24)$$

$$V_{\mathrm{T}k\Delta} = V_{\mathrm{T}k+} - V_{\mathrm{T}k-} \quad (25)$$

5.2. Experimental result of 2TX-1RX System

Experimental result of 2TX-1RX system is shown in fig. 11. Efficiency becomes maximum when $V_{\mathrm{T2}}/V_{\mathrm{T1}}$ approaches to $L_{\mathrm{m2}}/L_{\mathrm{m1}}$, and receiving power at fixing maximum TX voltage is increased when R_{L} became larger, similarly to the theory.

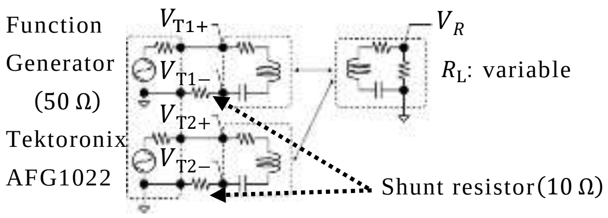


Fig. 10 Experimental setup of 2TX-1RX system

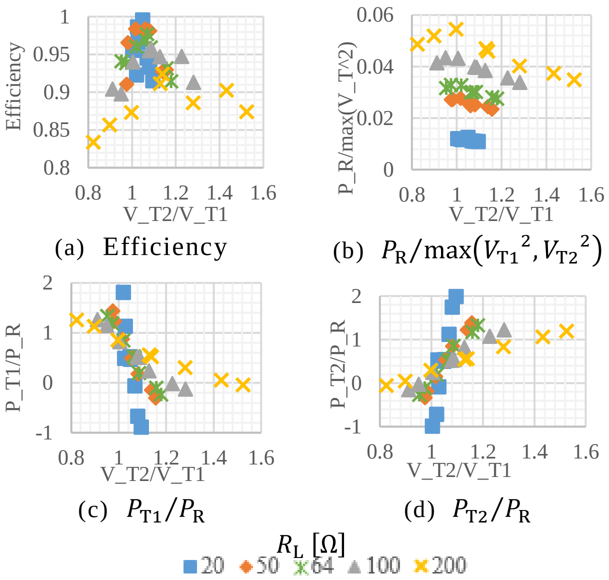


Fig. 11 Experimental result of 2TX-1RX system

Also, TX transmitting power becomes negative when voltage rate is far from mutual inductance rate. This result indicates that decreasing efficiency or higher requirement for TX rated power occurs when TX voltage rate is far from mutual inductance rate.

6. Conclusion and future works

In this paper, we investigated MTMR-WPT. Firstly, requirement of maximum efficiency for MTMR-WPT was derived using transform formula to STSR-WPT. Secondly, proposal and detail requirement for maximizing efficiency and numerical calculation result of 2TX-1RX and 1TX-2RX system were demonstrated. Effect of $R_{\mathrm{L1,2}}$ and $V_{\mathrm{T2}}/V_{\mathrm{T1}}$ to efficiency and P_{R} were investigated. From the theory and calculation results, load impedances should be the same value, and TX voltage rate should be in proportion to mutual inductance value for maximizing efficiency. Thirdly, one case of experimental result of 2TX-1RX system was shown, and verified that it followed the theory. Also, it was proved reverse flowing of power to a certain TX was occurred when $V_{\mathrm{T2}}/V_{\mathrm{T1}}$ differed more than about 10% from $L_{\mathrm{m2}}/L_{\mathrm{m1}}$. We are going to investigate on experiment of other cases and maximizing efficiency control of NTX-1RX system.

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