

# Compensation of Excessive Angular Momentum in a Re-Adhesion Control of an Electric Train

Takafumi Koseki, Takafumi Hara

Department of Electrical Engineering and Information Systems, The University of Tokyo, Bunkyo-Ku Tokyo, Japan  
[takafumikoskeki@ieee.org](mailto:takafumikoskeki@ieee.org)

**Abstract**—A smart dynamic control for suppressing slip between rails and wheels is a key technology for a good traction performance in electric railways. An appropriate choice of the timing of increasing traction torque after a temporary torque reduction to mitigate slip is significant for a successful dynamic adhesion control. We propose to monitor excessive angular momentum of a wheel to decide the timing of re-enhancement of the wheel torque. Numerical and experimental results verify advantage of the proposed control scheme.

**Keywords**—slip, slip speed, electric traction, adhesion control, angular momentum

## I. INTRODUCTION

The best use of adhesion force in electrical trains is always significant technical issue for precise and fast train operation. A theoretical basis for determining an appropriate timing of re-raising traction torque after a temporary torque reduction for recovering adhesion is requested for such a smart re-adhesion control. Therefore, various re-adhesion controls were intensively studied by electrical and control engineers for electric railways. Conventionally, many engineers tried empirical tuning based on slip velocity and angular acceleration methods[1][2]. There are proposals to use information on tangential load torque from a disturbance observer [3][4] recently. This idea has applied to a commercial rolling stock as described in [5]. Kawamura describes experimental implementation of an estimator of adhesion force and load torque[6]. However it is still difficult to guarantee re-adhesion[7].

We propose to think of physical meaning of re-adhesion and to monitor “excessive angular momentum”[8][9][10] calculated from tangential load torque and slip velocity in this paper. For establishing the theoretical basis, single 1-axle & 1-truck model will be studied in details, first of all. We will also try practical extension to 4-axle & 2-truck model through numerical calculations. We will also describe a fundamental experimental verification by monitoring and estimating transient state variables through a vehicle test.

TABLE I. NOMENCLATURE

Symbol	Value	Unit
$J_R$	Rotary inertia moment around wheel axis	kg · m <sup>2</sup>
$J$	Equivalent inertial moment of translational total mass	kg · m <sup>2</sup>
$\omega_w$	Driving wheel angular speed	rad/s
$G_r$	Mechanical gear ratio	--

$M_R$	Equivalent translational inertia of a rotary wheel set	kg
$M$	Translational body mass per one axle	kg
$v_b$	Longitudinal body speed	m/s
$v_s$	Slip speed	m/s
$\omega_s$	Slip angular speed	rad/s
$T_L$	Tractive torque	Nm
$r$	Radius of a wheel	m
$F_{ext}$	External running resistance force	N

## II. PROPOSAL OF RE-ADHESION CONTROL METHOD

### A. Excessive torque and excessive angular momentum

Introduction of “excessive torque” and “excessive angular momentum”

Dynamic equations of wheel and vehicle in a 1-axle & 1-truck model are as follows:

$$J_R \dot{\omega}_w = G_r T_m - T_L \quad (1)$$

$$M \dot{v}_b = \frac{T_L}{r} - F_{ext} \quad (2)$$

$$v_s = r \omega_w - v_b \quad (3)$$

$$\omega_s = \omega_w - \frac{v_b}{r} \quad (4)$$

$$\mu = \frac{T_L}{W g r} \quad (5)$$

where the constants and variables in the formula are summarized in Table 1. The tractive coefficient  $\mu$  has been introduced here in (5). The tractive coefficient  $\mu$  is the tangential force divided by the normal force at a rail-wheel contact point. The maximal tractive coefficient  $\mu_{MAX}$  represents its largest amount.

The following discussion will be based on the coordinates of rotary motion around an axle. The equivalent inertia moments  $J_R = \frac{1}{4} M_R r^2$  and  $J = M r^2$  are calculated by assuming rectangular solid form of a train body and disk form of a wheel. From these relationship, the slip speed is derived as follows

$$\dot{\omega}_s = \frac{1}{J_R} \left\{ G_r T_m - \left( 1 + \frac{J_R}{J} \right) T_L + \frac{J_R}{J} r F_{ext} \right\}. \quad (6)$$

The excessive torque is defined as follows:

$$T_{ex} = J_R \dot{\omega}_s = G_r T_m - \left(1 + \frac{J_R}{J}\right) T_L + \frac{J_R}{J} r F_{ext}. \quad (7)$$

Since we cannot know exact actual amount of the external resistance force, we can only calculate the following amount  $T_{ex-fault}$  obtained from subtracting tangential load torque from motor torque as follows:

$$T_{ex-fault} = G_r T_m - \left(1 + \frac{J_R}{J}\right) T_L. \quad (8)$$

By integrating (7), we can theoretically define the “excessive angular momentum” as follows:

$$L_{ex} = \int \left\{ G_r T_m - \left(1 + \frac{J_R}{J}\right) T_L + \frac{J_R}{J} r F_{ext} \right\} dt, \quad (9)$$

where we can just calculate

$$L_{ex-fault} = \int \left\{ G_r T_m - \left(1 + \frac{J_R}{J}\right) T_L \right\} dt \quad (10)$$

exactly in practice. We need a certain additional physical assumption to the external running resistance for calculating (9). By using the excessive torque and excessive angular momentum of a wheel, the slip angular speed and slip angular acceleration are calculated as follows respectively:

$$\omega_s = \frac{1}{J_R} \int T_{ex} dt = \frac{1}{J_R} L_{ex} \quad \text{and} \quad (11)$$

$$\dot{\omega}_s = \frac{1}{J_R} T_{ex}. \quad (12)$$

These formula means that the excessive angular moment shall be suppressed as small as possible for a quick recovery of adhesion. In the other words,  $L_{ex}$  can be an useful index to know the slip situation and to control the recovery of adhesion.

TABLE II. DEFINITION OF THE TIMES OF INCIDENTS

Time when slip starts	$t_{slip}$
Time when a slip is detected	$t_{detect}$
Time when the motor torque is reduces	$t_{Tdown}$
Time when motor torque is re-raised	$t_{Tup}$
Time from the slip detection through the torque reduction	$\tau_1$
Time from the torque- reduction through re-raise	$\tau_2$

Fig. 1 shows the relationship between transient change of the excessive angular momentum and a slip & re-adhesion control. Fig. 1(a) shows the transient behaviour of the motor torque and tangential load torque. Figs. 1 (b) and (c) show the excessive torque and excessive momentum, respectively. Table 2 explains the different times used in Fig. 1. When a slip occurs, the tangential load torque  $T_L$  becomes small and  $T_{ex-fault}$  (and  $T_{ex}$  become positive. To change these amount to negative, the motor torque must be reduced by re-adhesion controller. When the  $L_{ex}$  is converged to zero, the adhesion is recovered. For calculating the integration of  $T_{ex}$  the initial

value of  $L_{ex}$  is set to zero, when the calculation is started before the large slip starts.

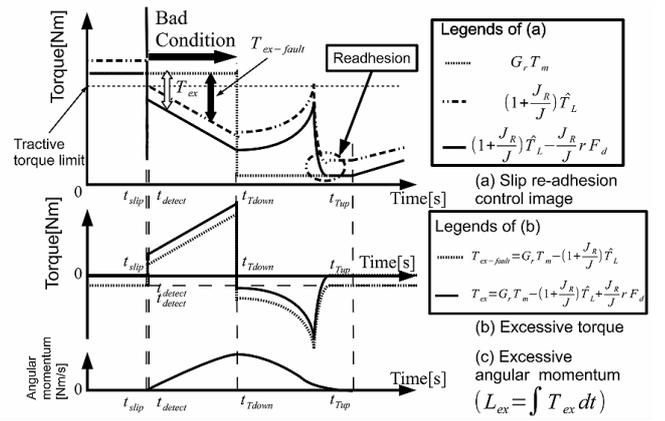


Fig. 1. Schematic view of excessive angular momentum during a re-adhesion control

## B. Estimation of torque corresponding to tangential force

The tangential load torque  $T_L$  must be estimated for the calculations of  $T_{ex}$  and  $L_{ex}$  above. By using the following full-order disturbance observer

$$\frac{d}{dt} \begin{bmatrix} \hat{\omega}_w \\ \hat{T}_L \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{J} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\omega}_w \\ \hat{T}_L \end{bmatrix} + \begin{bmatrix} G_r \\ J_R \\ 0 \end{bmatrix} T_m + H(\hat{\omega}_w - \omega_w), \quad (13)$$

the estimated tangential load torque  $\hat{T}_L$  is obtained, by assuming zero-order dynamics of the load torque, where  $H$  in (13) is the observer gain. The output equation of the system is

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\omega}_w \\ \hat{T}_L \end{bmatrix}. \quad (14)$$

## C. Torque commands for a re-adhesion control

As you see in (7) and (9), the estimation of external contribution  $\left(\frac{J_R}{J}\right) r F_{ext}$  is needed for calculating the excessive torque and angular momentum. Here we assume that the external resistance force  $F_{ext}$  is not changed during the re-adhesion control and identical to its initial amount before the slip occurs in practice.

For simple discussion on the 4-axle & 2-truck model, this term  $\left(\frac{J_R}{J}\right) r F_{ext}$  will be neglected fore the theoretical study in the next section.

Another simplification will be introduced in the calculation of the excessive angular momentum: The curve of the excessive torque changes linearly and the integral calculation is approximated by the calculation of a rectangular area as shown in Fig. 2. The approximate rectangular area is always larger than precisely calculated excessive momentum; this is a conservative estimation in a real-time control.

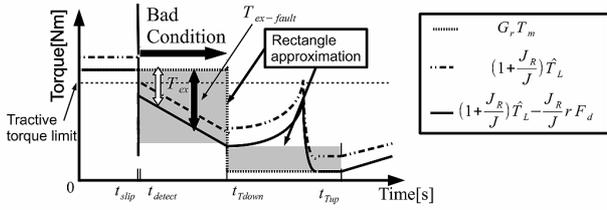


Fig. 2. A rectangular approximation of the excessive angular momentum  $L_{ex}$ .

### 1) Application to 4-axle & 2-truck model

The re-adhesion control in previous section was based on the 1-axle & 1-truck model. This control is extensively applied to 4-axle & 4-truck model in this section. Fig. 3 shows general traction configuration of a electric train. Fig. 3 (a), (b) and (c) show a 1-inverter & 4-motor system, a set of two 1-inverter & 2motor systems, and set of four 1-inverter & 1-motor system, respectively. The 4-axle & 2-truck model in Fig. 3(b) will be investigated in this chapter as a case of multiple motors connected to an inverter for simplicity, although the type in Fig. 3 (a) is usual in real application presently.

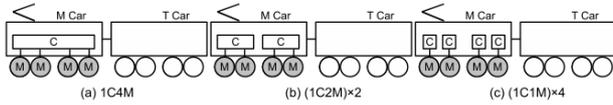


Fig. 3. Traction system of a train to be investigated.

In this system, the following information is available for the re-adhesion control:

- (1) average torque of the two motors from measured average motor current,
- (2) average angular speed of the two axle  $(\omega_1 + \omega_2)/2$ , and
- (3) Tangential load torque estimated from the average angular speed and the average motor torque.

If one axle is in slip and another axle is in adhesion, the average tangential load torque is larger than the tangential torque in slip. On the other hand, the average tangential load torques are almost same, if two axle are simultaneously in slip. The re-adhesion control in the case that one of two axle is in slip is more difficult. The following discussion will deal with such a difficult case.

- (1) The motor torque command must be reduced smaller than the smallest tangential load torque around the slipping axle, in spite that only average load torque can be estimated.
- (2) The excessive angular momentum must be compensated at the axle in the worst situation.

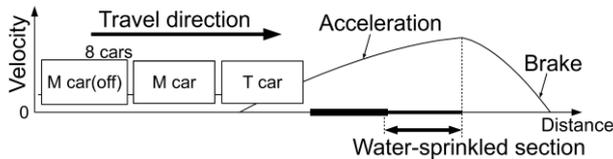


Fig. 4. An experimental case for a re-adhesion control test.

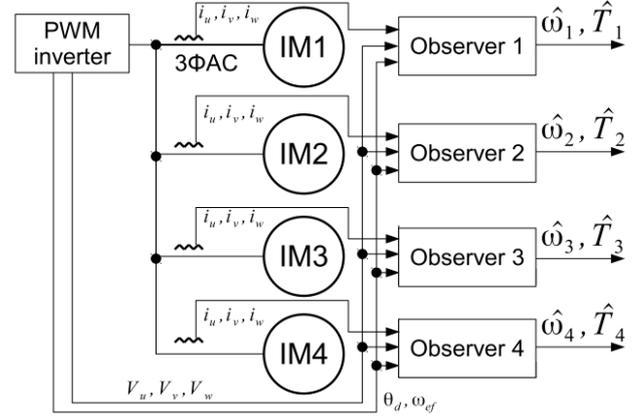


Fig. 5. Traction control system with four traction motors with individual current sensors.

Therefore an empirical factor  $k$  for deciding the amount of torque reduction is introduced here based on experimental results of a vehicle test using conventional re-adhesion controls[2] as follows. Fig. 4 and Fig. 5 show the test field and sensor configuration of the vehicle test.

Fig. 6 shows the measured results in case I, *i.e.*, under a dry track surface condition, and Fig. 7 shows corresponding measured results in case II, *i.e.*, under a wet track surface condition. The motor torque is calculated from measured motor current at each axle, and the tangential load torque is calculated from the disturbance observer in (13) using motor current and angular speed of each axle. It is supposed that the both axles are in adhesion and in the dry case in Fig. 6, and the “uncorrected” excessive torque  $L_{ex-fault}$  at the first axle is almost identical to the average “uncorrected” excessive torque  $L_{ex-fault}$  in Fig. 6. Fig. 7 shows that the “uncorrected” excessive torque  $L_{ex-fault}$  at the slippery first axle is often larger than the average “uncorrected” excessive torque  $L_{ex-fault}$ , but almost smaller than three times of the average “uncorrected” excessive torque  $L_{ex-fault}$ . Therefore the empirical factor  $k$  was set to 3.

### 2) Excessive torque from a start of slip through torque reduction

When the slips of multiple axles start, the first axle starts to slip first of all. Fig. 8 shows the behaviour of the motor torque and the tangential load torque when only the first axle is slipping. We assume the two conditions for our re-adhesion control.

- (1) Since the second axle is in adhesion, the motor torque and the tangential load torque are almost identical in the second axle, *i.e.*, the excessive torque at the second axle is almost zero.
- (2) The average excessive torque seen from the inverter controller,  $T_{ex,\tau_1}$ , is a half of the one at the first slipping axle based on the assumption above.

Consequently, the excessive angular momentum  $L_{ex,\tau_1}$  is calculated as follows:

$$L_{ex,\tau_1} = 2\tau_1 T_{ex,\tau_1} \cdot \quad (15)$$

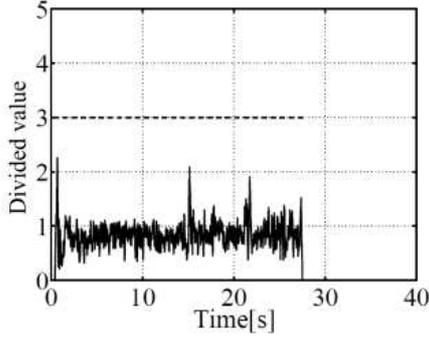


Fig. 6. Excessive torque of the axle 1 divided by the average excessive torques of four axles: Case I

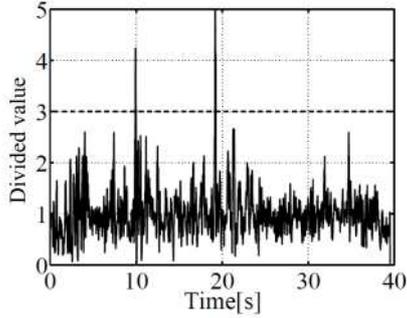


Fig. 7. Excessive torque of the axle 1 divided by the average excessive torques of four axles: Case II

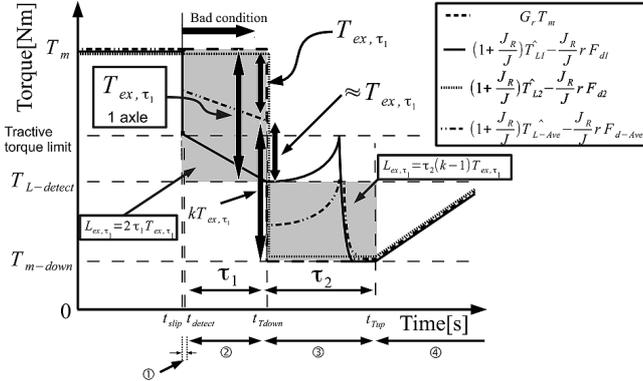


Fig. 8. Schematic view of the proposed re-adhesion control (when running resistance is neglected.)

### 3) Excessive torque from the torque reduction through a torque re-raising

Since we watch the compensation of the excessive angular momentum at the first axle, the excessive angular momentum  $L_{ex,\tau_2}$  from the motor torque reduction through the motor torque re-raising is as follows:

$$L_{ex,\tau_2} = \tau_2(k-1)T_{ex,\tau_1} \quad (k > 1) \quad (16)$$

### 4) Time $\tau_2$ between the torque-reduction and the re-raising

The time between the from the motor torque reduction through the motor torque re-raising is derived from the conservation of excessive angular momentum, (15) and (6) as follows:

$$2\tau_1 T_{ex,\tau_1} = \tau_2(k-1)T_{ex,\tau_1} \quad (k > 1) \quad (17)$$

$$\tau_2 = \frac{2}{k-1} \tau_1 \quad (18)$$

### 5) Fundamental torque control pattern

The motor torque command  $T_m^*$ , which guarantees the re-adhesion is described by using  $T_m$ , the motor torque before slip started,  $T_{L-detect}$ , the tangential load torque when the motor torque was reduced,  $T_{m-down}$ , the reduced motor torque as follows:

$$T_m^* = \begin{cases} T_m & (t_{slip} < t < t_{Tdown}, (1) \& (2)) \\ T_{m-down} & (t_{Tdown} \leq t \leq t_{Tup}, (3)) \\ T_{M-down} + G_{up}(T_m - T_{m-down})(t - t_{Tup}) & (t_{Tup} < t, (4)) \end{cases} \quad (19)$$

Fig. 8 and Table III show the fundamental pattern of the torque control procedure. The reduced amount of the motor torque  $T_{m-down}$  at  $t_{slip} < t < t_{detect}$  shall be decided to set finally the excessive angular momentum  $L_{ex}$  to zero as follows:

$$L_{ex,\tau_2} = \tau_2(T_{L-detect} - T_{m-down}) \quad (20)$$

TABLE III. PROCEDURE OF TORQUE CONTROL ACTIONS

Time	Status and action
$t_{slip}$	Large slip occurrence
(1)	Infinitesimal detection time $T_m^* = T_m$
$t_{detect}$	Slip detection
(2)	$T_m^* = T_m$ for $\tau_1$
$t_{Tdown}$	Torque reduction
(3)	$T_m^* = T_{m-down}$ for $\tau_2$
$t_{Tup}$	Starting torque re-raise
(4)	Gradually re-raising torque with time-coefficient of $G_{up}$

## III. NUMERICAL CASE STUDY OF A RE-ADHESION CONTROL OF A CAR WITH MULTIPLE INDUCTION MOTORS

### A. Modeling of a rolling-stock and rail surface

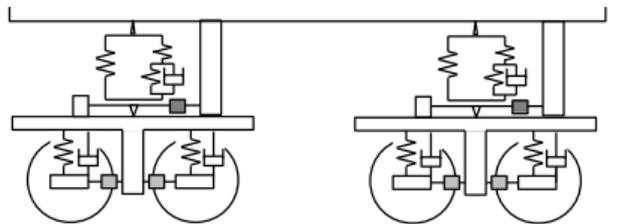


Fig. 9. Schematic view of 4-axle and 2-truck model

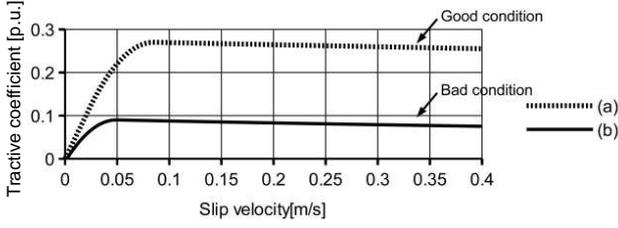


Fig. 10. Two cases of adhesion coefficient and slip velocity profile: (a)  $\mu_{max} = 0.27$ , and (b)  $\mu_{max} = 0.09$

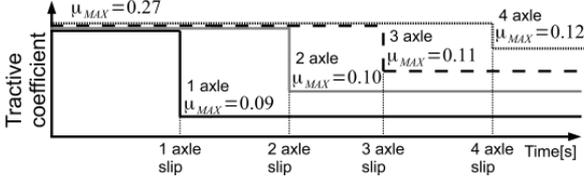


Fig. 11. A scenario of a numerical case study on changes of adhesion condition.

The model for the numerical case study is shown in Fig. 9, the motor specification of which is summarized in Table IV. The following four conditions are assumed:

- (1) the body is laterally and longitudinally symmetric,
- (2) the rail is straight and parallel,
- (3) lateral motion dynamics is neglected, and
- (4) mechanical braking force is neglected.

The mechanical model in Fig. 9 is studied for considering dynamic change of axle weights and truck oscillations. Also the tractive coefficient is assumed to be a unique function of microscopic slip speed[8] as shown in Fig. 10. Fig. 11 shows the change of the maximal tractive coefficient of each wheel in the scenario of the numerical study. The vertical gradient is assumed to be constant 0.3%.

TABLE IV. SPECIFICATION OF THE ASYNCHRONOUS TRACTION MOTOR FOR THE NUMERICAL CASE STUDY

Primary resistance	$5.8 \times 10^{-2} \Omega$
Primary inductance	21mH
Main inductance	20mH
Secondary inductance	21mH
Secondary resistance	$5.3 \times 10^{-2} \Omega$
Pole-pair number	3
Nominal power	160kW

The scenario of the numerical case is summarized in Table V.

TABLE V. CASE STUDY SCENARIO

Time [s]	Actions
0	Start acceleration $\alpha = 3$ [km/h/s], $\mu_{MAX} = 0.27$ in Fig. 13 (a)
6	Reduction of adhesion coefficients $\mu_{MAX} = 0.09, 0.10, 0.11, 0.12$
15	Recovery of the adhesion coefficients to $\mu_{MAX} = 0.27$
24	Simulation ends

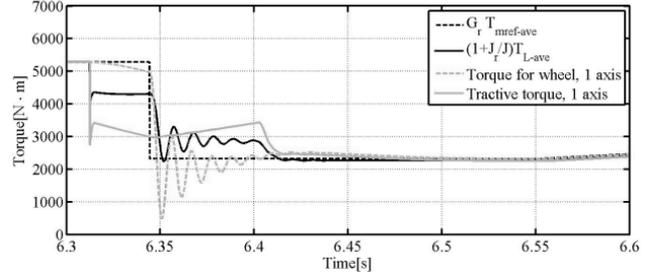


Fig. 12. Motor torque and tractive torque at axle 1.

## B. Numerical results and discussions

The empirical factor is set to  $k=3$  and the time  $\tau_2$  between the motor torque reduction to re-raising has been automatically decided by the conservation of the excessive angular momentum. The reduced motor torque command is set smaller than the smallest axle torque and the re-adhesion control is consequently successful in Fig. 12.

## IV. EXPERIMENTAL VERIFICATION

Motor currents and motor angular speed from vehicle tests at Shinkeisei Line in April 2009 as shown in Fig. 3-5 have been analyzed in order to verify the usefulness of the monitoring the excessive angular momentum to know actual adhesion situation. It is also expected that we can analyze the problems of the state estimation based on the measurement of the average current of four motor currents.

Figs. 13 show transient behaviour of motor torque command, angular inverter frequency (speed), and real motor angular speed of a motor. The re-adhesion control worked between 6 and 7 sec. in Fig. 13; this re-adhesion control was successful. Another re-adhesion control between 18.5 and 20.5 sec. was unsuccessful in Fig. 13. The blue curve A in Fig. 14 shows the amount of uncorrected excessive angular momentum  $L_{ex-fault}$  in (10). The pink curve B in Fig. 14 shows the integrated value of the third term of the right hand side of (7), i.e., the contribution of the external running resistance to the calculation of the excessive torque  $T_{ex}$ , by assuming this external contribution is not changed during a re-adhesion control and identical to the amount just before the slip started. That is to say, the amount obtained by subtracting B from A represents the excessive angular momentum in (9), which is the index to diagnose the adhesion proposed in this paper. Fig. 14 shows that the excessive angular momentum after  $t=6.3$ sec. was kept negative, which indicates a successful re-adhesion control. On the other hand, Fig. 15 shows that the excessive angular momentum became once negative at  $t=18.9$ sec., however, it turned positive at  $t=20.2$ sec. Again. It suggests that the re-adhesion control was finally unsuccessful. These experimental results show the usefulness to monitor the excessive angular momentum represented in (9) for diagnosing the adhesion condition and deciding an appropriate timing for re-raising motor torque in a re-adhesion control.

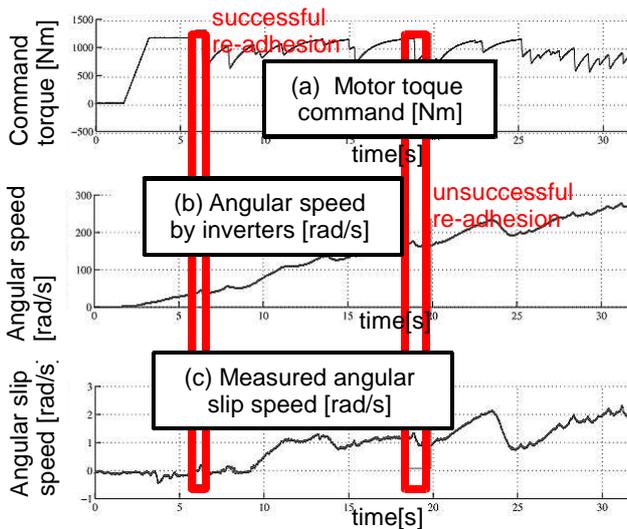


Fig. 13. Measured transient response of re-adhesion controls: (a) command torque, (b) angular speed calculated in an inverter, (c) real angular slip speed at an axis.

## V. CONCLUSIONS

We have proposed to monitor the excessive torque and the excessive angular momentum for a re-adhesion control of an electric train. We have also shown an approximate simplified calculation of the excessive angular momentum based on practically acceptable physical assumption for simplicity. Fundamental control strategy and transient behaviour of the proposed control has been studied in detail through theoretical and numerical analyses based on single axle-single truck model. The proposed simplified monitor of the excessive angular momentum has the following advantages.

(1) Real-time computational load for the calculation of the excessive angular moment can be sufficiently small by the simple rectangular approximation of the excessive torque waveform.

(2) Wrong slip detections can be avoided by checking the approximate excessive angular momentum after a certain waiting time during reduced motor torque.

We can also conclude the following facts by calculating case studies of multiple axle drives.

(3) The control strategy can also be applicable to the practical cases, where an inverter supplies electric power to two motors in spite of small deterioration of performance indexes.

Small difference of wheel radii results in the difference of torque distribution between two axles. However, the proposed re-adhesion control is sufficiently robust to practically possible difference of the wheel radii. The usefulness to monitor the excessive angular momentum in re-adhesion control has experimentally verified through a vehicle test. Further technical discussion shall be extended to a case study of a combination of a single inverter and four motors in future.

## ACKNOWLEDGMENT

The authors thank Mr. Y. Okada at Mitsubishi Electric Corp., Mr. K. Hisatomi at Shinkeisei Railway Corp., and Prof. K. Kondo for their friendly supports in technical discussions and the experimental verification through the vehicle tests at Shinkeisei line in April 2009.

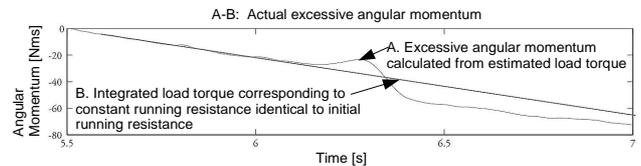


Fig. 14. Measured excessive angular momentum in case of a successful re-adhesion control

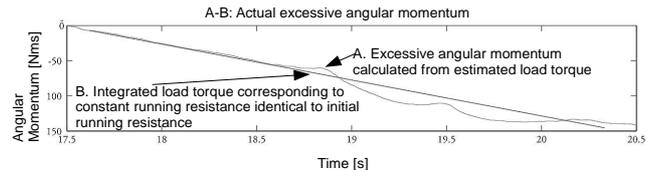


Fig. 15. Measured excessive angular momentum in case of an unsuccessful re-adhesion control

## References

- [1] Technical committee of adhesion control technology investigation in railway system: "Adhesion control technology in railway vehicle", IEEJ technical report No.673, (1998) (in Japanese)
- [2] H. Iida, S. Kita, M. Kumano, and T. Kikuchi: "Development of Fuzzy Adhesive Control System", Proc. the 1995 National convention record, I.E.E.Japan Industry Applications Society, Vol.2, pp. 269-272, (1995) (in Japanese)
- [3] O. Kiyoshi, O. Yasuaki, N. Ken, M. Ichiro, Y. Shinobu: "An Approach of Anti-slip Re-adhesion Control of Electric Motor Coach Based on First Order Disturbance Observer", IEEJ Trans. IA, Vol.120-D, No.3, pp.382-389 (2000-3) (in Japanese)
- [4] S. Kadowaki, K. Oishi, I. Miyashita, S. Yasukawa: "Anti-Slip Re-adhesion Control of Electric Motor Coach(2M1C) Based on Disturbance Observer and Speed Sensor-less Vector Control", IEEJ Trans. IA, Vol.121-D, No.11, pp.1192-1198 (2001-11) (in Japanese)
- [5] S. Kadowaki, T. Hata, H. Hirose, K. Oishi, N. Iida, M. Takagi, T. Sano, S. Yasukawa: "Application Results and Evaluation of Anti-slip Re-adhesion Control Based on Speed Sensor-less Vector Control and Disturbance Observer for Electric Multiple Units, Series 205-5000", IEEJ Trans. IA, Vol.124-D, No.9, pp.909-916 (2004-9) (in Japanese)
- [6] H. Ohshita, A. Kawamura: "Measurement of Tractive Coefficient during Acceleration and Deceleration with the tractive force Tester and Maximum Adhesion Controls". IEEJ, IIC, IC-07-80, pp. 59-64 (2007) (in Japanese)
- [7] K. Nagase, N. Tagawa, E. Maebashi, H. Nomoto, K. Okikura: "The realities of adhesion between rail wheels on main line (result of the survey by slipping adhesive bogie)", Proc. Japan Society of Mechanical Engineers the paper collection, Vol.504-C, pp. 282-286 (1988 / 8) (in Japanese)
- [8] Y. Tsukinokizawa, T. Koseki, H. Negoro, A. Murahashi: "A calculation of the trajectory of tractive adhesion coefficient and readhesion possibility during readhesion control of an electric rolling stock", IEEJ, TER, TER-08-37, pp.45-50 (2008 / 9) (in Japanese)
- [9] T. Hara, T. Koseki: "Study on re-adhesion control by monitoring excessive angular momentum in electric railway tractions", The 12th International Workshop on Advanced Motion Control (AMC 2012), Sarajevo, Bosnia and Herzegovina, March 25th-27th, 2012
- [10] T. Hara, T. Koseki, Y. Okada and K. Hisatomi: "Re-Adhesion Control in Asynchronous Motor Drives for an Electric Train by Monitoring Excessive Angular Momentum," IEEJ Trans. on IA, Vol. 133, No. 9, pp. 909-916, (2013-09) (in Japanese).