# Extending the Resolution Range of the Cascaded Generalized Inverse

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***Abstract*— By far the most popular method of resolving redundant systems involves application of the 2-norm-optimizing Moore-Penrose pseudo-inverse. Pseudo-inverse resolution resolves systems uniquely, continuously, and most notably with simple implementation for general systems. However, resolution using 2-norm fails to exploit general systems' full realizable output space when input bounds are present. The Cascaded Generalized Inverse (CGI) was introduced to address this problem by iteratively reallocating oversaturated input resolutions to other, under-utilized inputs. In this paper, it is demonstrated that although successful in extending the resolution range of 2-norm, CGI still fails to capture the full benefit of a system's realizable output space. To remedy this, an Extended CGI (eCGI) is introduced to allow a larger space of successful resolution through selective desaturation of saturated inputs through CGI. eCGI is demonstrated in resolving kinematic redundancy and was shown in a numerical example to increase the maximum controllable end-effector acceleration over what is possible with CGI to the full physically achievable space.**

## I. Introduction

Redundancy is a useful characteristic which is often employed in systems operating in dynamic, unpredictable environments, in areas where maintenance is inconvenient, and in applications in which failure cannot be tolerated. While redundancy offers some useful control strategies, a complication arises in allocation of task contributions when no particular problem is to be solved by these redundant degrees of freedom.

The problem of redundancy is considered as follows: The matrix $\boldsymbol{B} \in \Re^{mxn}$ translates input variables $\boldsymbol{u} \in \Re^{n}$ to output space values $\boldsymbol{v} \in \Re^{m}$ as follows:

$$\boldsymbol{v} = \boldsymbol{B}\boldsymbol{u} \tag{1}$$

If the number of inputs is equal to the number of outputs ($n = m$) then we say the system is deterministic, and the required selection of inputs to produce a desired output can be uniquely determined by inverting the relationship in (1).

If, however, the number of inputs exceeds the number of outputs ($n > m$) the system is deemed redundant. An ideal way to deal with this is to eliminate the system redundancy by posing additional constraints improving system performance [1]. If there is no clear way to eliminate the redundancy however, we can select an infinite number of possible input combinations to yield a given desired output.

How to select among these infinite possible choices has been the subject of a great deal of deliberation, but the majority of implementations have made use of some variant of the Moore-Penrose pseudo-inverse which resolves redundant systems as:

$$\boldsymbol{u} = \boldsymbol{B}^{\dagger}\boldsymbol{v} \tag{2}$$

where if $\boldsymbol{B}$ is full row rank, the pseudo-inverse, $\boldsymbol{B}^{\dagger}$, of $\boldsymbol{B}$ is defined as:

$$\boldsymbol{B}^{\dagger} := \boldsymbol{B}^{T}(\boldsymbol{B}\boldsymbol{B}^{T})^{-1} \tag{3}$$

Functionally, the pseudo-inverse resolves systems by optimizing the 2-norm of resolved inputs, or

$$\min(\sqrt{u_1^2 + u_2^2 + ... + u_n^2}) \tag{4}$$

subject to (1). However, pseudo-inverse resolution is valued far more for its analytical tractability than for its physical interpretation. Pseudo-inverse resolution resolves systems uniquely, continuously, and foremost with a very simple implementation for general systems.

Psuedoinverse resolution does however come with a drawback: a 2-norm optimal resolution does not necessarily fall within maximum input bounds, and as such pseudo-inverse resolution is unable to exploit a system's full realizable output space.

Alternative resolution schemes have been proposed which do not suffer from this drawback, such as infinity-norm resolution [2], [3], which minimizes the maximum input contribution subject to the task constraint (1) or

$$\min\left(\max\left(|u_1|, |u_2|, ..., |u_n|\right)\right) \tag{5}$$

But thanks to pseudo-inverse resolution's simplicity in implementation compared to these other methods, the majority of implementations opt to sacrifice this output space in favor of using 2-norm optimization in resolving redundancy.

Rather than switching resolution methods entirely, applications in which this lost output space is a significant concern typically make use of methods to extend the resolution range of 2-norm. The largest extension of 2-norm resolution currently available is given by the Cascaded Generalized Inverse (CGI) [4] which iteratively reallocates input contributions resolved in excess of their limits to other, underutilized inputs.

In this paper it will be shown that, although successful in extending the resolution range of 2-norm, CGI resolution still fails to exploit the full resolution range of general systems. A method of extending the Cascaded Generalized Inverse will be proposed and demonstrated in resolving the problem of kinematic redundancy.

## II. Extending 2-norm Resolution

### A. Least Squares with Clipping

The simplest method of extending 2-norm, Least Squares with Clipping [5] simply evaluates the system using 2-norm and truncates inputs resolved in excess of their bounds. Although very easy to implement, Least Squares with Clipping will fail to produce the desired output, $\boldsymbol{v}$. Consequently it is not typically recommended to actually operate in this clipping region, and if entered, corrective action should be taken to safely return to pseudo-inverse resolution capability.

### B. Redistributed Pseudo-inverse

The Redistributed Pseudo-inverse [6] is a simple procedure which succeeds in extending the resolvable range of 2-norm, for a period, without sacrificing fulfillment of the desired output, $\boldsymbol{v}$. The Redistributed Pseudo-inverse can be described by the following three-step process:

1) Evaluate the system using 2-norm resolution, as shown in (2), (3).
2) Truncate any inputs in excess of their bounds:

$$|u^{\dagger}_{i_1}| > u_{i_1\max}, |u^{\dagger}_{i_2}| > u_{i_2\max}, ..., |u^{\dagger}_{i_k}| > u_{i_k\max} \implies \tag{6}$$

$$\text{if } u^{\dagger}_{i_j} > u_{i_j\max},\ u^{\text{redist.}}_{i_j} = u_{i_j\max} \tag{7}$$

$$\text{if } u^{\dagger}_{i_j} < -u_{i_j\max},\ u^{\text{redist.}}_{i_j} = -u_{i_j\max} \tag{8}$$

3) Re-resolve non-truncated inputs using 2-norm:

$$\boldsymbol{u}'_{i_1...i_k} = (\boldsymbol{B}'_{i_1...i_k})^{\dagger}(\boldsymbol{v} - \sum_{\gamma=1}^{k} \boldsymbol{B}_{i_\gamma} u_{i_\gamma}) \tag{9}$$

Where $\boldsymbol{B}'_{i_1.i_2...i_k}$ is the matrix $\boldsymbol{B}$ with the $i_1$, $i_2$, ... , and $i_k$'th column removed and $\boldsymbol{u}'_{i_1.i_2...i_k}$ is the vector $\boldsymbol{u}$ with the $i_1$, $i_2$, ..., and $i_k$'th element removed.

This Redistributed Pseudo-inverse does successfully extend resolution beyond what is possible with simple 2-norm resolution. However, as the re-resolution is carried out using 2-norm optimization, the full possible output space of these reoptimized inputs in general cannot be captured. That is, a 2-norm optimal re-resolution may yield an unfeasible resolution regardless of whether an alternative feasible re-resolution exists.

### C. Cascaded Generalized Inverse

The Cascaded Generalized Inverse (CGI) [4] is the natural solution to this problem of lost output space in the Redistributed Pseudo-inverse. The Redistributed Pseudo-inverse was introduced to "redistribute" input allocations when 2-norm yields unrealizable input values, by truncating and re-resolving the remaining inputs. CGI simply repeats this process in the event the re-resolution step of the Redistributed Pseudo-inverse yields unrealizable input values, that is:

1) Evaluate the system using 2-norm resolution, as shown in (2),(3).
2) Truncate any inputs in excess of their bounds.
3) Re-resolve non-truncated inputs using 2-norm.
4) Repeat steps 2 and 3 until a feasible result is obtained or until less than $m$ inputs remain unsaturated.

If less than $m$ inputs (the dimension of the desired output) remain to be re-resolved, CGI will fail to yield a correct resolution for an arbitrary desired output, $\boldsymbol{v}$. Under such a circumstance alternative corrective methods such as output magnitude scaling must be used to correct for resolution failure. It is its main benefit that CGI will always fail in resolution at an output magnitude greater than or equal to 2-norm.

Variations exist where one variable is arbitrarily saturated in each iteration in the interest of speed [7], and the "most" saturated variable is saturated each iteration to preserve directionality at the expense of speed [8].

An intuitive variation of CGI is depicted in Fig. 1. This variation proceeds only so long as a successful resolution can be achieved, by continuing until the number of inputs remaining is less than the degree of the output.

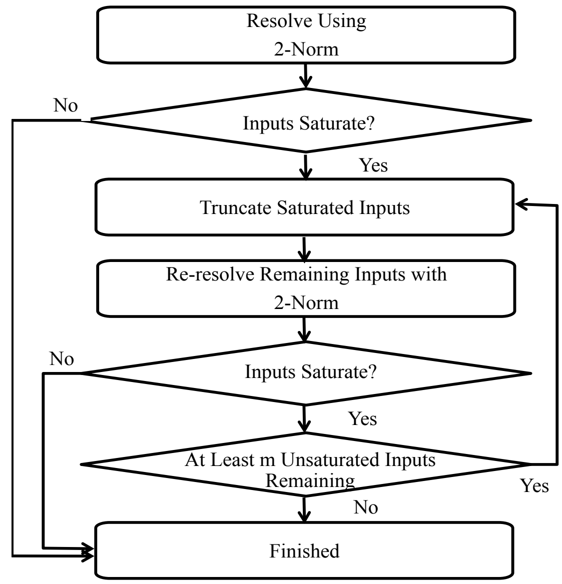


Fig. 1. Flowchart of Cascaded Generalized Inverse redundancy resolution.

The Cascaded Generalized Inverse currently represents the largest extension of 2-norm resolution and has been applied to many problems including aircraft control allocations [9], [10], VTOL control systems [11], ship berthing [12], and ship positioning systems [13].

### D. Continuous Cascaded Generalized Inverse

In [14] we demonstrated that although successful in extending the resolution range of 2-norm, both the Redistributed Pseudo-inverse and the Cascaded Generalized Inverse lose the characteristic of continuous resolution offered by 2-norm. Frequently through the CGI process of truncation and redistribution, input assignment weighting changes or even reverses direction in re-resolution. If a lower level truncation occurs after a weighting change, discontinuities can result in resolution.

Such discontinuity would no doubt raise significant dynamic concerns, should they surface in some systems. To overcome this discontinuity we introduced a variation of CGI, termed Continuous CGI (cCGI). cCGI is functionally equivalent to CGI resolution, but a restriction is placed on the resolution domain such that only one input per resolution level (with the initial 2-norm being one level, and every subsequent truncation and re-resolution being another level) be permitted to saturate. If multiple inputs saturate in a single resolution level, resolution fails. Although in general cCGI resolution does not allow for the same resolution range as CGI resolution, resolution using cCGI is guaranteed to be continuous. Additionally in physical systems, restriction against multiple simultaneous saturations is not so restrictive as re-resolved inputs tend to increase saturation rate. This makes single level simultaneous saturations uncommon.

Applications where this discontinuity is not a serious concern or applications which have been tested sufficiently to ensure discontinuity does not surface in resolution will still continue to make use of CGI due to its extended output space. In the sections to follow, it will be demonstrated that these applications which prioritize this larger output space of CGI may still not be getting the full potential out of their systems. An extension of the CGI method (and with no modification an extension of cCGI) will be proposed allowing greater output space resolution.

## III. Output Space Limitations in CGI Resolution

Although CGI represents the largest currently available extension of 2-norm, it does not necessarily extend resolution to the maximum capability of the resolved system. In this section, through a numerical example, it will be demonstrated that CGI fails to extend resolution to the maximum potential system space.

The system to be considered is resolution of a 4-link, kinematically-redundant planar manipulator. Kinematic redundancy is a useful characteristic utilized in robotics, endowing systems with such useful behaviors as obstacle avoidance [15], joint limit avoidance [16], and fault tolerance. As a redundant system, however, kinematically redundant manipulators have a complication in translation from a desired task-space output (end-effector trajectory) to joint space inputs. This problem of kinematic redundancy is a specific but well-known case of (1) and is formulated as follows: The Jacobian matrix, $\boldsymbol{J}$, relates joint velocities, $\dot{\boldsymbol{q}}$, to end-effector velocity, $\dot{\boldsymbol{x}}_{\text{eff}}$ as

$$\dot{\boldsymbol{x}}_{\text{eff}} = \boldsymbol{J}\dot{\boldsymbol{q}} \tag{10}$$

where

$$[\boldsymbol{J}]_{(i,j)} = \frac{\delta[\boldsymbol{x}_{\text{eff}}]_i}{\delta \boldsymbol{q}_j} \tag{11}$$

and $\boldsymbol{x}_{\text{eff}} \in \Re^m$ is the vector of end-effector position coordinates and $\boldsymbol{q}$ is the vector of joint positions.

Our system is a four-link arm operating in 2-dimensional space as seen in Fig. 2. Therefore there exists two degrees of joint-space redundancy which can be freely assigned. Two main approaches are taken towards resolving this redundancy.

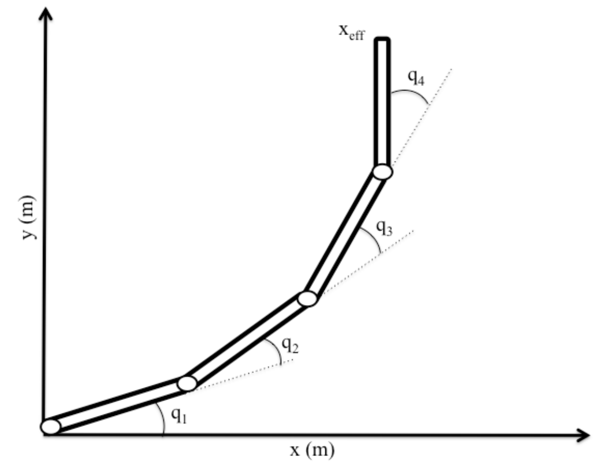


Fig. 2. 4-link planar arm used in demonstration of CGI resolution output range restriction.

The first option, which is analytically tractable, is direct resolution in the velocity space [17]. As (10) is a direct linear relationship, it can be resolved by applying the pseudo-inverse of $\boldsymbol{J}$ as seen in (2). However, velocity-space redundancy resolution neglects dynamics and motor-limitations. The resolved joint velocities are time-differentiated to yield the corresponding acceleration values, which are finally dynamically compensated with the expectation the motor will be able to supply sufficient torque.

The alternative then is resolution in the acceleration space [18], which allows dynamics and maximum motor exertions to be factored directly into the redundancy resolution. Acceleration-domain redundancy resolution can be performed by differentiating (10) with respect to time, yielding:

$$\ddot{\boldsymbol{x}}_{\text{eff}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} \tag{12}$$

and solving simultaneously with the system dynamics:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{13}$$

where $\boldsymbol{M} \in \Re^{nxn}$ is the manipulator inertia matrix, $\boldsymbol{C} \in \Re^{nx1}$ is the vector of Coreolis and centrifugal force terms, $\boldsymbol{g} \in \Re^{nx1}$ is the vector of gravity terms, and $\boldsymbol{\tau} \in \Re^{nx1}$ is the applied joint torque.

In the following example, we will be resolving the kinematic redundancy in the acceleration domain. For the following example and for all simulations to follow, let Configuration A represent the following arm states:

Configuration A:
$\boldsymbol{q} = [\frac{\pi}{32}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}]$ (rad)
$\dot{\boldsymbol{q}}(t=0) = [0,0,0,0]$ (rad/sec)
$\boldsymbol{\tau}_{\text{max}} = [5,1,1,1]$ (Nm)

For this example, we will consider the arm to be in configuration A. We are free to make $\tau_{1_{\text{max}}}$ larger than the other motors, since motor 1 would be mounted on the base and therefore would not contribute to increased

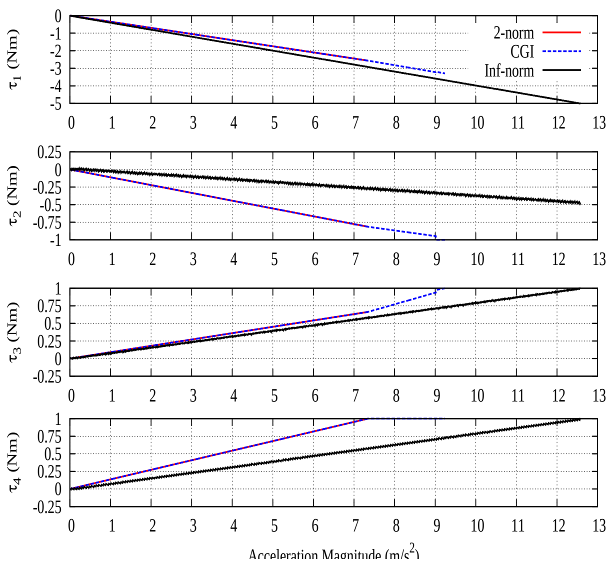


Fig. 3. Resolution of joint torques for given acceleration magnitude using 2-norm, CGI, and infinity-norm resolution. CGI is seen to successfully extend the realizable output over 2-norm, but not to the level possible using infinity-norm resolution.

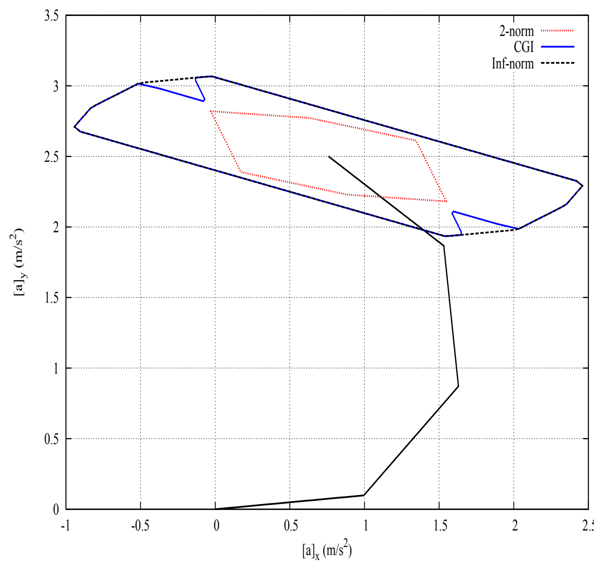


Fig. 4. Scaled maximum end-effector acceleration achievable using 2-norm, CGI resolution, and infinity-norm resolution. CGI is seen to extend resolution range of 2-norm, but not to the full system capability represented by infinity-norm.

arm inertia. Since we are not interested in the dynamics of the arm and are only focused on the limitations of the redundancy resolution, Coreolis forces will be neglected and the manipulator inertia matrix $\boldsymbol{M}$ will be considered as the identity matrix. The arm will be considered horizontally-planar, so gravity forces can also be neglected.

Fig. 3 illustrates a static resolution of the kinematic redundancy in this arm. From configuration A, the arm is tasked with a desired end-effector acceleration magnitude in the direction $\theta = 335$ degrees, and redundancy resolution is carried out using 2-norm, CGI, and infinity-norm resolution to determine an appropriate set of joint torques. It can be seen that CGI and 2-norm resolve the system equivalently until $\|a\| \approx 7$ (m/s$^2$) at which point 2-norm fails resolution due to $\tau_4$ being resolved in excess of its maximum. CGI, however, continues to resolve the system until $\|a\| \approx 9$ (m/s$^2$) by truncating oversaturated inputs and redistributing their contributions. Infinity-norm, on the other hand, allocates torque in constant proportion and independently of the allocations seen in either 2-norm or CGI (as infinity-norm and 2-norm optimizations are independent and CGI is based on the 2-norm). Infinity-norm resolution is seen to increase the maximum realizable acceleration of this system to over 12 (m/s$^2$), which is guaranteed by the definition of the infinity-norm to be the maximum system-capable acceleration.

Fig. 4 illustrates the maximum end-effector acceleration (scaled down by a factor of 10) possible in all directions in configuration A.

## IV. Proposal: Extended CGI

The problem of limited output space in CGI resolution can be rephrased as a problem of CGI saturating inputs inconsistently with those saturated by infinity-norm, due to the difference in optimization criteria between 2-norm and infinity-norm. As earlier discussed, an infinity-norm solution chooses a resolution that minimizes the largest input contribution. Therefore, if a feasible solution exists, resolution using infinity-norm will always yield a feasible result.

Although entirely discrete in terms of formulation and numerical resolution, it can be said that a 2-norm resolution and an infinity-norm resolution are certainly related by the physical characteristics of the system. Optimization using 2-norm is highly related to minimization of system energy [19], and it grows as the square of any particular input. So while a 2-norm optimization will not likely be equivalent to the infinity-norm optimal value, a 2-norm resolution is unlikely to be a particularly bad selection based on the infinity-norm criteria.

This characteristic can be observed through the previously conducted comparison of 2-norm, CGI, and infinity-norm in Fig. 3. Upon the start of resolution, 2-norm and infinity-norm both begin distributing torques with the same rough scheme. The same motors are assigned positive torque and the same motors are assigned negative torque in both, with some redistribution of torques to satisfy the corresponding optimization criteria. When CGI fails resolution, it has successfully saturated two of the three same inputs and with the same sign as the infinity-norm resolution.

In our particular example, how best to exploit this relationship is apparent. After CGI fails, we can desaturate $\tau_2$ (which is not saturated in the infinity-norm optimization) and maintain saturation of $\tau_1$ and $\tau_3$ (which coincide with the infinity-norm optimized inputs) and re-resolve this new deterministic system. The result will be a linear, continuous extension bridging the CGI resolution at its failure point to

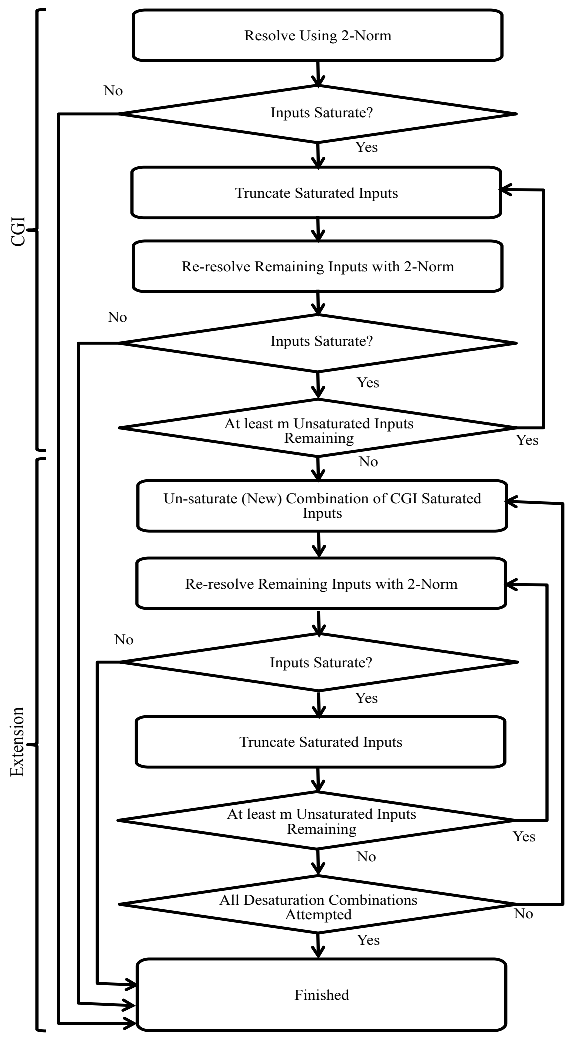


Fig. 5. Flowchart of Extended Cascaded Generalized Inverse redundancy resolution.

the infinity-norm solution at the maximum system capability point.

This approach is simple to generalize. When a system is to be resolved, the full CGI process is carried out, as seen in Fig. 1. After CGI completes, if resolution is unsuccessful, combinations of inputs should be selectively desaturated and resolution can be attempted again using the CGI process of truncation and re-resolution using the pseudo-inverse. If re-evaluation after desaturating one combination of inputs is unsuccessful, a different combination is tried until a feasible result is obtained or until all possible desaturation combinations have been attempted. A schematic of this approach can be seen in Fig. 5. This approach toward resolution will be termed as Extended CGI or eCGI.

## V. Simulations

### A. Static

Fig. 6 illustrates the simulation shown in Fig. 3 repeated using CGI, eCGI, and infinity-norm resolution. CGI is seen

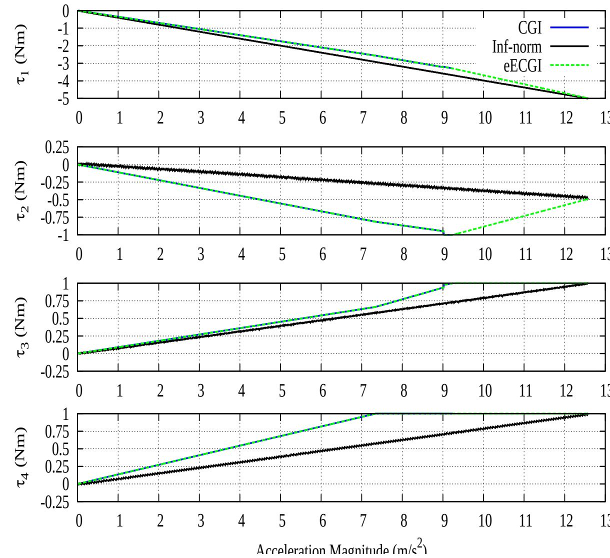


Fig. 6. Resolution of joint torques for given acceleration magnitude using CGI, eCGI, and infinity-norm resolution. eCGI is seen to successfully extend the realizable output from CGI to the full system capability.

to stop resolution at $\|a\| \approx 9$ (m/s$^2$) after saturating $\tau_4$, $\tau_2$, and $\tau_3$, in that order. eCGI then successfully extends CGI resolution to infinity-norm resolution (both in acceleration magnitude and to the unique infinity-norm resolution at the system boundary) by desaturating $\tau_2$ and re-resolving $\tau_2$ and $\tau_1$ subject to saturated $\tau_3$ and $\tau_4$.

### B. Output Space

Fig. 7 illustrates the simulation shown in Fig. 4 repeated using 2-norm, CGI, eCGI, and infinity-norm resolution. eCGI is seen to extend the maximum controllable end-effector acceleration from that possible with CGI to that possible with infinity-norm, which is by definition the maximum system capability.

## VI. Discussion

The current state of eCGI demands that all input desaturation combinations be attempted, with no preference given to combinations which may be more likely to yield higher output resolutions. A great deal of application tailoring potential is therefore available, allowing the system designer to determine which inputs combinations should be selected first and how far the desaturation attempts ought to be taken. For example, the system in Section V only requires individual input desaturation implementation to yield the full infinity-norm system capability. This knowledge allows for a decrease in the amount and complexity of code to implement eCGI. A useful, though not necessary, step in implementation of eCGI would be determination of which desaturation combinations most likely to allow for extension of the output space.

Additionally as can be seen in Fig. 7, there will no doubt be many applications in which eCGI is capable of extending 2-norm to the infinity-norm capable output space. This will

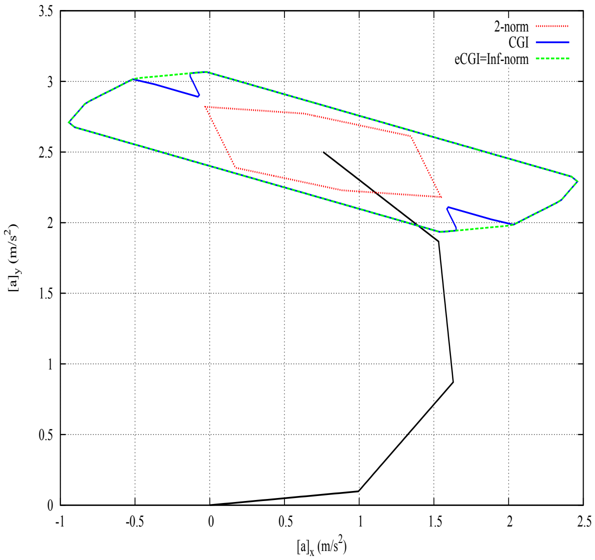


Fig. 7. Scaled maximum end-effector acceleration achievable using 2-norm, CGI resolution, and eCGI resolution. Here, eCGI resolution can be seen extending the feasible output space to the full potential space.

make eCGI instrumental in the construction of a more generalized version of the 2-norm/Infinity-norm switching system proposed in [20].

## VII. Conclusion

The Cascaded Generalized Inverse (CGI) is a popularly utilized method of redundancy resolution which was introduced to extend the range of resolvable outputs of the ubiquitous pseudo-inverse approach to redundancy resolution. In this paper we showed that although successful in its goal of extending the 2-norm, CGI fails to exploit the full resolution range of systems. In this paper, we introduce the Extended Cascaded Generalized Inverse (eCGI), which allows for extension of CGI through desaturation of saturated inputs in CGI. Two main conclusions can be drawn with regard to this method:

1) eCGI is the largest available extension of 2-norm resolution, allowing fuller utilization of system capabilities than any preceding 2-norm based method.
2) CGI has the potential for discontinuity in resolution. Therefore eCGI as an extension of CGI has the potential for discontinuity in resolution. Like CGI, applications should be chosen either in which the effects of this discontinuity will not be substantial should they surface, or in which through testing the possibility of discontinuity has been disregarded.

In this paper eCGI was demonstrated in resolving torque redundancy in a 4-link redundant manipulator. It was shown that in some directions CGI was unable to resolve physically realizable accelerations and consequently experienced significant resolvable output nonhomogeneity in some directional neighborhoods. When eCGI was applied, the full physically achievable output range was rendered feasible.

Future work includes experimental application of eCGI in improving operating speed of redundant manipulators, evaluation of a continuity condition for eCGI in extending cCGI resolution, as well as application of eCGI in realizing a more general version of the 2-norm/infinity-norm switching system.

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