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2-norm, Infinity-norm Continuous Switching Resolution in Biarticularly Actuated Robot Arms

Travis Baratcart, Valerio Salvucci, and Takafumi Koseki
Department of Electrical Engineering and Information Systems, The University of Tokyo, Japan
b_travis@koseki.t.u-tokyo.ac.jp, valerio@koseki.t.u-tokyo.ac.jp, takafumikoseki@ieee.org

***Abstract*—In the resolution of redundant systems, two popular resolution schemes, optimization using 2-norm and infinity-norm, have been commonly employed. However, each finds its greatest utility in resolution in very different circumstances. The 2-norm optimizes in analog to minimizing system energy (sometimes by putting large strain on individual actuators). Likewise, the infinity-norm minimizes individual actuator contributions, at the expense of total system efficiency. As such, it would be preferable to utilize a resolution system which optimizes along the 2-norm while actuator levels are low and of little concern, and switches to infinity-norm when individual actuator levels grow too large. Although previously hypothesized, this paper marks the first successful realization of the described resolution scheme and is proposed for use in resolving biarticular actuation redundancy— a biologically inspired actuation structure enabling effective force transfer, stiffness regulation, and homogeneous force output. To realize this switching system, a resolution scheme called the Cascaded Generalized Inverse (CGI) is used to enable continuous switching between the two norms at a pre-defined switching threshold. Proof of continuity is demonstrated, and implementation is simulated.**

I. Introduction

Redundancy in systems allows for increased dexterity and robustness for systems operating in dynamic applications, but its use implies a complication in the control structure which must be resolved.

The problem of redundancy is considered as follows: The matrix $\boldsymbol{B} \in \Re^{mxn}$ translates actuation variables $\boldsymbol{u} \in \Re^n$ to task space values $\boldsymbol{v} \in \Re^m$ as follows:

$$\boldsymbol{v} = \boldsymbol{B}\boldsymbol{u} \tag{1}$$

If $n > m$ the system is said to be redundant, and if in a nonsingular configuration, there exist an infinite number of inputs, $\boldsymbol{u}$, which can achieve a desired output, $\boldsymbol{v}$. This problem of selecting the "best" resolution has been the subject great deliberation.

Such redundant systems are typically resolved through utilization of the Moore-Penrose pseudo-inverse [1] which resolves the criteria function while optimizing the 2-norm, i.e.

$$\min\left(\sqrt{u_1^2 + u_2^2 + \ldots + u_n^2}\right) \tag{2}$$

subject to (4). 2-norm optimization is useful due to its simple implementation (closed form for general systems), uniqueness, and continuity of resolution. Also, as a minimization of a sum of squares a parallel can be drawn between 2-norm optimization and a minimization of energy in the system.

However, the 2-norm does not always represent an ideal resolution given physical constraints. As the pseudo-inverse optimizes through minimization of an average of inputs, it does not explicitly concern itself with the individual inputs. Consequently, 2-norm can often place a large burden on an individual input, with the goal of total system efficiency.

To combat this, many researchers have instead recommended optimization using the infinity-norm [2], [3], which resolves the criteria function such that the largest input is minimized, or

$$\min\left(\max\left(|u_1|, |u_2|, \ldots, |u_n|\right)\right) \tag{3}$$

Resolution using the infinity-norm effectively protects inputs from over-exertion by only assigning large input effort when absolutely necessary. However, when resolved inputs fall well within safe operating conditions, infinity-norm resolution does not efficiently allocate input contributions.

It would be preferable then, to evaluate with 2-norm (minimizing energy) when resolved inputs are sufficiently low and only switch to infinity-norm when inputs grow too large. Such a switching system was hypothesized by [4], but has yet to be realized due to the fact that 2-norm and infinity-norm resolutions rarely switch continuously and that infinity-norm closed form solutions are not well developed.

This paper introduces the first realized 2-norm/infinity-norm switching system, which is implemented in resolving biarticular actuation redundancy.

Biarticular actuators, actuators which act on two joints simultaneously are a biologically inspired structure which allow for such benefits as force transfer from proximal to distal joints (enabling reduction of limb inertia) [5], feedforward stiffness control [6], and improved homogeneity of force and stiffness output with respect to direction [7]. Further, as a standard biarticular implementation can be described by a static, low order system, it is one of the only redundant systems with a developed closed-form infinity norm resolution [8] (one other solution exists redundant EV lateral dynamic control [9]). Such a closed form solution allows for an assurance of continuity of our proposed switching system.

In order to realize the described switching system, a resolution scheme called the Cascaded Generalized Inverse (CGI) will be utilized. CGI was introduced [10] to extend the realizable output range of the 2-norm, due to 2-norm's failure to consider input bounds. It will be shown that, in our application, CGI resolution extends 2-norm to the full

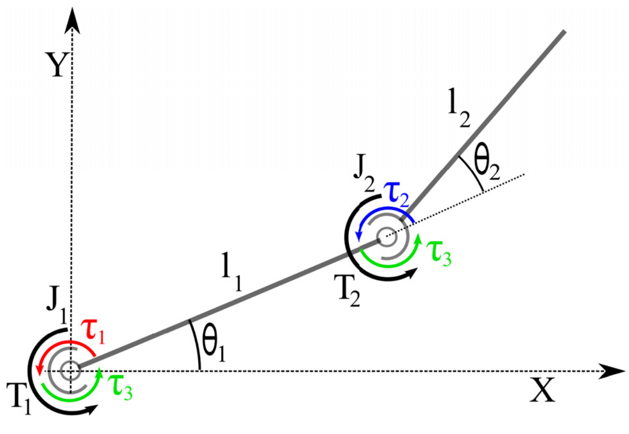

Fig. 1. Model of biarticular actuation in human arm

realizable extent of the system (within input bounds τ_{switch}), enabling continuous switching with infinity-norm.

II. Background

Fig. 1 illustrates a simplified model of actuation in the human arm [7]. This problem of redundancy in this system is formulated as follows:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \boldsymbol{B}\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{4}$$

Where T_1 and T_2 are output joint torques, τ_1 and τ_2 are the monoarticular actuator torques about joints 1 and 2 respectively, and τ_3 is the biarticular actuator torque actuating both joints.

2-norm resolution of (4) yields

$$\tau_1 = \frac{2}{3}T_1 - \frac{1}{3}T_2 \tag{5}$$

$$\tau_2 = \frac{2}{3}T_2 - \frac{1}{3}T_1 \tag{6}$$

$$\tau_3 = \frac{1}{3}T_1 + \frac{1}{3}T_2 \tag{7}$$

If infinity-norm is used to resolve the system, the result [8] is

$$\tau_1 = \begin{cases} \frac{1}{2}(T_1 - T_2) & \text{if case 1} \\ T_1 - \frac{1}{2}T_2 & \text{if case 2} \\ \frac{1}{2}T_1 & \text{if case 3} \end{cases} \tag{8}$$

$$\tau_2 = \begin{cases} \frac{1}{2}(T_2 - T_1) & \text{if case 1} \\ \frac{1}{2}T_2 & \text{if case 2} \\ T_2 - \frac{1}{2}T_1 & \text{if case 3} \end{cases} \tag{9}$$

$$\tau_3 = \begin{cases} \frac{T_1}{2} + \frac{T_2}{2} & \text{if case 1} \\ \frac{1}{2}T_2 & \text{if case 2} \\ \frac{1}{2}T_1 & \text{if case 3} \end{cases} \tag{10}$$

where

$$\text{case } 1 := (T_1 \leq 0 \text{ and } T_2 \geq 0) \tag{11}$$

$$\text{or } (T_1 > 0 \text{ and } T_2 < 0) \tag{12}$$

$$\text{case } 2 := (T_1 \geq 0 \text{ and } T_2 \geq T_1) \tag{13}$$

$$\text{or } (T_1 < 0 \text{ and } T_2 < T_1) \tag{14}$$

$$\text{case } 3 := (T_2 \leq T_1 \text{ and } T_2 \geq 0) \tag{15}$$

$$\text{or } (T_2 > T_1 \text{ and } T_2 < 0) \tag{16}$$

To complete the switching system, a variant of CGI [11] which optimizes based on the "most saturated" input, will be utilized. This CGI variant resolves (4) as:

Case 0 :

$$\tau_1 = \tau_1^{\dagger} := \frac{2}{3}T_1 - \frac{1}{3}T_2 \tag{17}$$

$$\tau_2 = \tau_2^{\dagger} := \frac{2}{3}T_2 - \frac{1}{3}T_1 \tag{18}$$

$$\tau_3 = \tau_3^{\dagger} := \frac{1}{3}T_1 + \frac{1}{3}T_2 \tag{19}$$

Case $\tau_i(\text{a})$:

$$\tau_i = \tau_i^{\max} \tag{20}$$

$$\boldsymbol{\tau}_i' = (\boldsymbol{B}_i')^{-1}(\boldsymbol{T} - \boldsymbol{B}_i \tau_i^{\max}) \tag{21}$$

Case $\tau_i(\text{b})$:

$$\tau_i = -\tau_i^{\max} \tag{22}$$

$$\boldsymbol{\tau}_i' = (\boldsymbol{B}_i')^{-1}(\boldsymbol{T} + \boldsymbol{B}_i \tau_i^{\max}) \tag{23}$$

and define

$$\text{case 0 (2-norm)} := |\tau_i^{\dagger}| \leq \tau_i^{\max},\ i = 1,2,3 \tag{24}$$

$$\text{case } \tau_i(\text{a}) := \tau_i^{\dagger} > \tau_i^{\max},\ |\tau_i^{\dagger}| \geq |\tau_j^{\dagger}|,\ \forall j \neq i \tag{25}$$

$$\text{case } \tau_i(\text{b}) := \tau_i^{\dagger} < -\tau_i^{\max},\ |\tau_i^{\dagger}| \geq |\tau_j^{\dagger}|,\ \forall j \neq i \tag{26}$$

$$\tag{27}$$

Where $\boldsymbol{\tau}_i'$ is the vector $\boldsymbol{\tau}$ with the element τ_i removed, $\boldsymbol{B}_i$ is the i'th column of the matrix $\boldsymbol{B}$, and $\boldsymbol{B}_i'$ is the matrix $\boldsymbol{B}$ with the i'th column removed.

III. Proposal

In order to realize such a switching system, switching levels will be defined for the system, which describe the point at which the system should begin prioritizing motor protection. As asserted, while tasked with non-demanding torque output, the system will optimize in accordance with two-norm and minimize energy. Likewise, if tasked with large torque, the system will optimize using infinity-norm and minimize strain on individual motors. To complete the switching system, an intermediary region must be developed which allows continuous transitioning from low-demand to high-demand regions.

Defining all $\tau_i^{\max}$ as the desired switching level τ_{switch} in evaluation of CGI allows realization of the desired switching

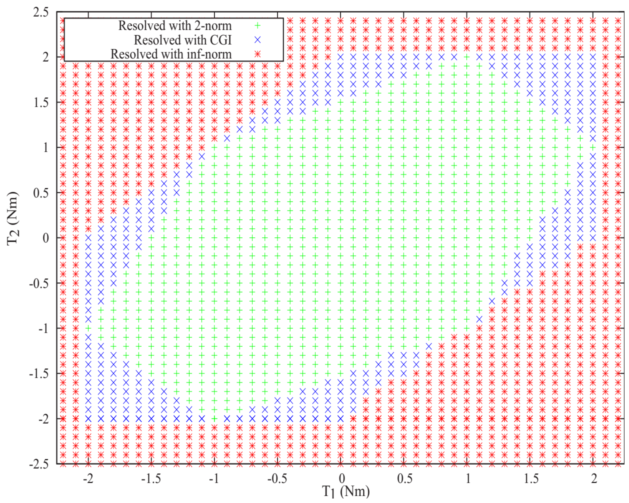

Fig. 2. Visualization of proposed switching system with unit switching torque. The system resolves with respect to 2-norm until an infinity-norm threshold is met, at which point torques are reallocated and further optimized with respect to the infinity-norm.

system as:

$$\boldsymbol{\tau} = \begin{cases} \boldsymbol{\tau}^{\dagger}, & \text{if } |\tau_i^{\dagger}| \le \tau_{\text{switch}},\ i = 1,2,3 \\ \boldsymbol{\tau}^{CGI}, & \text{if } \exists\, \tau_i^{\dagger}, \text{ s.t. } |\tau_i^{\dagger}| > \tau_{\text{switch}}, \\ & \text{and } |\tau^{CGI}| \le \tau_{\text{switch}},\ i = 1,2,3 \\ \boldsymbol{\tau}^{\infty}, & \text{if } \exists\, \tau_i^{CGI}, \text{ s.t. } |\tau_i^{CGI}| > \tau_{\text{switch}} \end{cases} \tag{28}$$

The model of the proposed switching system can be seen in Fig. 2

IV. Proof of Continuity of Switching System

In this section, the continuity of the proposed switching system will be demonstrated. CGI is defined such that it is continuous with increased torque magnitude (maintaining the ratios of T_1 and T_2, and is clearly continuous while re-optimized with respect to a single saturated motor torque. If discontinuity were to arise within CGI, it would occur while switching the input to be saturated and reoptimized against. Such switching events occur when two 2-norm torque resolutions simultaneously saturate.

To demonstrate CGI's continuity over the entire domain, we will show that at these simultaneous saturation events that the resolution with respect to both variables is equivalent. Further we will show that at such conditions, the 2-norm resolution is equal to the infinity-norm resolution (so these saturated variables cannot be reoptimized against). Note, this section of proof will concern itself with desired outputs s.t. $T_1, T_2 \neq 0$. A desired output with T_1 or T_2 equal to zero is a specific case which will be analyzed separately as part of the next section.

With CGI demonstrated as continuous with the infinity-norm at simultaneous saturation events, sequential saturation events will then be considered. It will be demonstrated that after 2-norm saturates and the other torques are reoptimized, when a subsequent motor torque saturates, the resolution is equal to the infinity-norm solution.

A. At least one saturated variable of 2-norm/CGI must be shared in the solution of the inf-norm

Analysis of the closed-form solutions of the 2-norm, CGI, and infinity-norm yield the following possible forms of resolutions for nonzero output torques:

	τ_1	τ_2	τ_3
$T_1 > 0$, $T_2 > 0$	$+$	$+$	$+$
	$+$	$-$	$+$
	$-$	$+$	$+$
$T_1 > 0$, $T_2 < 0$	$+$	$-$	$+$
	$+$	$-$	$-$

A useful characteristic of the infinity-norm is that its solution always has two or more inputs resolved as equal to the maximum value of the resolution.

Likewise, CGI fails to resolve after two or more inputs saturate. Although from the information given alone, it cannot be said that at this point the CGI resolution is equal to the infinity-norm solution, it can be said that at least one of the saturated variables is shared by both solutions.

Therefore the following can be said. If multiple variables saturate in 2-norm resolution and reoptimization with respect to niether results in increased output potential, the 2-norm resolution at this point is equal to the infinity-norm resolution. If a single variable saturates in 2-norm, CGI is used to reoptimize, and further reoptimization with respect to the variable which saturates in CGI resolution does not yield further output potential, the CGI resolution is equal to the infinity-norm resolution at that point. If reoptimization after CGI is possible, the resolution at the highest torque magnitude in that direction is equal to the infinity-norm resolution.

The remaining sections of proof will demonstrate:

1) Reoptimization after 2 simultaneous saturations, and reoptimization after CGI resolution are further unnecessary.
2) 2-norm is equal to infinity-norm during simultaneous saturations, and that in subsequent saturation events, CGI resolution is equal to infinity-norm.

Note, only the cases $(T_1, T_2 > 0)$ and $(T_1 > 0, T_2 < 0)$ will be considered. $(T_1, T_2 < 0)$ and $(T_1 < 0, T_2 > 0)$ are duals of these two cases.

B. Continuity at Equal Saturation Rates

1) $T_1, T_2 > 0$:

a) $\tau_1, \tau_2, \tau_3 \geq 0$:

- τ_1, τ_2 saturate simultaneously

$$\tau_1 = \frac{2}{3}T_1 - \frac{1}{3}T_2 = \tau^{\max} \tag{29}$$

$$\tau_2 = \frac{2}{3}T_2 - \frac{1}{3}T_1 = \tau^{\max} \tag{30}$$

$$\implies T_1 = T_2 \implies \tau_3 = 2\tau_1 = 2\tau^{\max} \tag{31}$$

Which is a contradiction, so this condition cannot actually occur.

- τ_1, τ_3 saturate simultaneously

$$\tau_1, \tau_3 = \tau^{\max} \implies T_1 = 2\tau^{\max} \tag{32}$$

Clearly T_1 can't be resolved higher than $2\tau^{\max}$, so here the 2-norm resolution is equal to the infinity-norm resolution.

- τ_2, τ_3 saturate simultaneously
  Dual of τ_1, τ_3 saturating simultaneously.

b) $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$:

- τ_1, τ_2 saturate simultaneously

$$\tau_1 = \frac{2}{3}T_1 - \frac{1}{3}T_2 = \tau^{\max} \tag{33}$$

$$\tau_2 = \frac{2}{3}T_2 - \frac{1}{3}T_1 = -\tau^{\max} \tag{34}$$

$$\implies T_1 = -T_2 \tag{35}$$

Which is a contradiction since both $T_1, T_2 > 0$, so this condition cannot actually occur.

- τ_1, τ_3 saturate simultaneously
  Dual of $\tau_1, \tau_2, \tau_3 \geq 0, \tau_1, \tau_3$ saturate simultaneously.
- τ_2, τ_3 saturate simultaneously

$$\tau_2 = -\tau^{\max}, \tau_3 = \tau^{\max} \tag{36}$$

$$\implies T_2 = 0 \tag{37}$$

$$\implies \tau_1 = \frac{2}{3}T_1 = 2\tau_3 = 2\tau^{\max} \tag{38}$$

Which is a contradiction, so this condition cannot actually occur.

c) $\tau_2, \tau_3 \geq 0, \tau_1 \leq 0$: Dual of $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$

2) $T_1 > 0, T_2 < 0$:

a) $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$:

- τ_1, τ_2 saturate simultaneously
  We note that either maximum τ_1 or maximum τ_2 (or both) must exist in the infinity norm solution in this configuration. We assume τ_1 is maximum in the infinity norm solution, and that if τ_1 and τ_2 saturate at a torque magnitude of k (corresponding to $T_1(k)$ and $T_2(k)$) that there is some realizable output with a larger magnitude $k + \Delta k$ in the same direction (whose resolution linearly approaches the infinity norm solution at the maximum attainable torque). At this point we resolve the system is resolved as

$$\tau_1 = \tau^{\max} \tag{39}$$

$$\tau_3 = T_1(k) + \Delta k T_1(k) - \tau^{\max} \tag{40}$$

$$\tau_2 = T_2(k) + \Delta k T_2(k) - T_1(k) - \Delta k T_1(k) + \tau^{\max} \tag{41}$$

$$= -\tau^{\max} + \Delta k T_2(k) - \Delta k T_1(k) \tag{42}$$

$$\implies \tau_2 < -\tau^{\max} \tag{43}$$

Which is a contradiction, so one cannot attain a higher magnitude output in the same direction with maximum τ_1. A similar process shows the same for τ_2. Therefore one cannot produce a higher magnitude torque in the same direction. This implies that the 2-norm solution is equal to the infinity norm solution at this point.

- τ_1, τ_3 saturate simultaneously
  Dual of $T_1, T_2, \tau_1, \tau_2, \tau_3 > 0$: τ_1, τ_3 saturate simultaneously.
- τ_2, τ_3 saturate simultaneously

$$\tau_2 = -\tau^{\max}, \tau_3 = \tau^{\max} \tag{44}$$

$$\implies T_2 = 0 \tag{45}$$

Which is a contradiction, so this condition cannot actually occur.

b) $\tau_1 \geq 0, \tau_2, \tau_3 \leq 0$: Dual of $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$.

C. CGI Equal to Inf-norm at Max. Realizable Torque

With the case of simultaneous saturation of 2-norm demonstrated to be equal to the infinity-norm, we will now demonstrate single input saturation in 2-norm, followed by saturation of another variable in CGI also yields an infinity-norm equivalent resolution. Let such notation as $\tau_1 \to \tau_2$ represent the condition that the left input saturates in 2-norm, the system is re-evaluated in CGI with respect to the left input, and the right input saturates in the CGI re-resolution.

1) $T_1, T_2 > 0$:

a) $\tau_1, \tau_2, \tau_3 \geq 0$:

- $\tau_1 \to \tau_2$

$$T_2 - (T_1 - \tau^{\max}) = \tau^{\max} \tag{46}$$

$$\implies T_1 = T_2 \tag{47}$$

$$\tag{48}$$

Therefore τ_1 and τ_2 saturate at the same time. This case was covered in section IV-B1a, and it was shown that it cannot actually occur.

- $\tau_2 \to \tau_1$
  Dual of $\tau_1 \to \tau_2$
- $\tau_1 \to \tau_3$, $\tau_2 \to \tau_3$, $\tau_3 \to \tau_1$, $\tau_3 \to \tau_2$
  Duals of $T_1, T_2, \tau_1, \tau_2, \tau_3 > 0$: Simultaneous saturation of τ_1 and τ_3.

b) $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$:

- $\tau_1 \to \tau_2$

$$T_2 - (T_1 - \tau^{\max}) = -\tau^{\max} \tag{49}$$

$$\implies T_1 - T_2 = 2\tau^{\max} \tag{50}$$

$$T_1 \leq 2\tau^{\max} \implies \tau_2 \leq 0 \tag{51}$$

Which is a contradiction, so this condition cannot actually occur.

- $\tau_2 \to \tau_1$
  Dual of $\tau_1 \to \tau_2$
- $\tau_1 \to \tau_3$
  Dual of $T_1, T_2, \tau_1, \tau_2, \tau_3 > 0$: Simultaneous saturation of τ_1 and τ_3.
- $\tau_2 \to \tau_3$

$$\tau_3 = T_2 + \tau^{\max} = \tau^{\max} \tag{52}$$

$$\implies T_2 = 0 \tag{53}$$

Which is a contradiction. Therefore, this condition cannot occur.

- $\tau_3 \to \tau_2$
  Dual of $\tau_2 \to \tau_3$

c) $\tau_2, \tau_3 \geq 0, \tau_1 \leq 0$*:* Dual of $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$

2) $T_1 > 0, T_2 < 0$*:*

a) $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$*:*

- $\tau_1 \to \tau_2,\ \tau_2 \to \tau_1$
  Dual of $T_1 > 0$, $\tau_1, \tau_3 \geq 0,\ T_2 < 0,\ \tau_2 \leq 0$: τ_1 and τ_2 saturate simultaneously
- $\tau_1 \to \tau_3,\ \tau_3 \to \tau_1$
  Dual of $T_1, T_2 > 0,\ \tau_1, \tau_2, \tau_3 \geq 0$: Simultaneous saturation of τ_1 and τ_3.
- $\tau_2 \to \tau_3,\ \tau_3 \to \tau_2$
  Dual of $T_1, T_2 > 0,\ \tau_1, \tau_3 \geq 0,\ \tau_2 \leq 0,\ \tau_2 \to \tau_3$.

b) $\tau_2, \tau_3 \leq 0, \tau_1 \geq 0$*:* Dual of $\tau_1, \tau_3 \geq 0, \tau_2 \leq 0$

3) $T_1 > 0, T_2 = 0$*:* In this case, 2-norm resolves the system as

$$\tau_1 = \frac{2}{3} T_1 \tag{54}$$

$$\tau_2 = -\frac{1}{3} T_1 \tag{55}$$

$$\tau_3 = \frac{1}{3} T_1 \tag{56}$$

τ_1 will saturate at $T_1 = \frac{3}{2}\tau^{\max}$ and larger torques will be reevaluated by CGI as

$$\tau_1 = \tau^{\max} \tag{57}$$

$$\tau_2 = -(T_1 - \tau^{\max}) \tag{58}$$

$$\tau_3 = T_1 - \tau^{\max} \tag{59}$$

Both τ_2 and τ_3 will saturate when $T_1 = 2\tau^{\max}$. Since all three torques are saturated, this CGI resolution is equal to the infinity-norm solution at this point.

4) Other Cases: $(T_1 < 0, T_2 = 0), (T_1 = 0, T_2 > 0)$, and $(T_1 = 0, T_2 < 0)$ are duals of $(T_1 > 0, T_2 = 0)$.

$(T_1, T_2 = 0)$ is a trivial case.

V. Simulations

A. Static Simulation

Fig. 3 demonstrates the proposed switching system in resolving the statics of a biarticular arm. The arm is tasked with increasing magnitude torque T such that $T_1 = T_2 = T$ and the redundancy is resolved using the proposed switching system. Switching levels are set at unit torque. The resolution is seen to be a piecewise continuous waveform composed of three linear segments corresponding to 2-norm, CGI, and infinity-norm resolution.

B. Path Simulation

To demonstrate the switching systems efficacy in resolving biarticular actuation in dynamic conditions, a 2-link arm in the standard biarticular configuration introduced in Sec. I is commanded along a trajectory and redundancy is resolved using the proposed switching system. The end-effector is commanded to move in straight line motion from an initial position with joint positions $\boldsymbol{\theta}^{\text{init}} = [\frac{\pi}{4}, \frac{\pi}{4}]$ to an end position with joint positions $\boldsymbol{\theta}^{\text{fin}} = [\frac{\pi}{3}, \frac{\pi}{3}]$. To ensure continuity of acceleration, a cubic spline was utilized to generate the trajectory taking

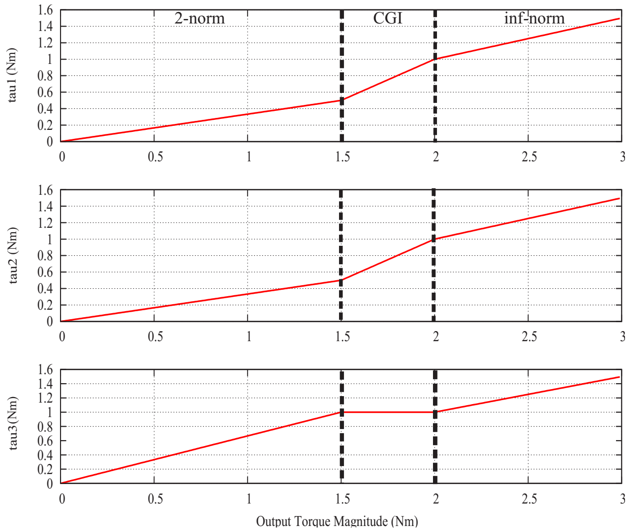

Fig. 3. Switching system resolution of manipulator in single position with increasing desired output torque. Composed of three piece-wise linear regions corresponding to 2-norm, CGI, and inf-norm.

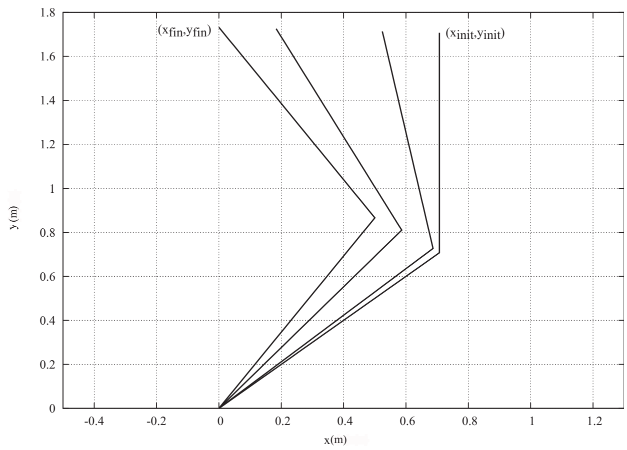

Fig. 4. Arm trajectory using switching redundancy resolution.

the manipulator to and from rest in 1.2 seconds. Switching levels are all set as unit torque and the inertia matrix is set as the identity matrix. Coriolis and gravity terms are neglected as they won't effect evaluation of the redundancy resolution. The described trajectory is illustrated in Fig. 4.

Fig. 5 illustrates the torques seen at each motor across the described trajectory. It is seen that resolution successfully switches continuously through 2-norm, CGI, and infinity-norm as the corresponding conditions are met. Fig. 6 illustrates the 2-norm and infinity-norm of the resolved torques of the proposed switching system and compares them with the corresponding 2-norms and infinity-norms seen at the same positions (along the switching-resolution trajectory) utilizing 2-norm and infinity-norm resolution. Along this trajectory, it is found that the switching system has a maximum 2-norm reduction of approximately 6 percent with respect to

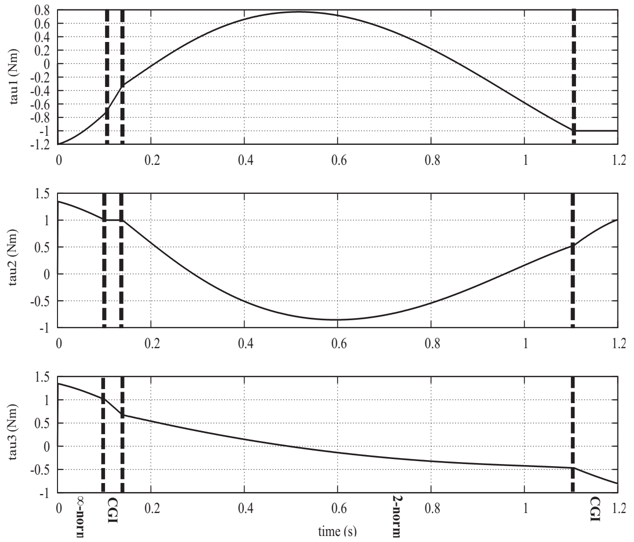

Fig. 5. Torque resolution of switching system whle following trajectory. Resolution passes through all three resolution regions, continuously.

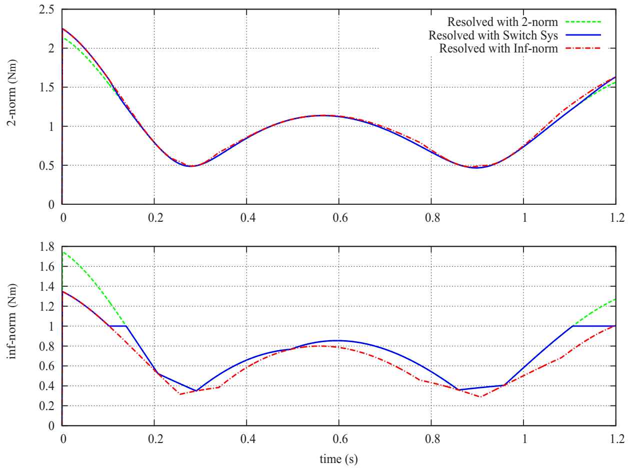

Fig. 6. 2-norm and infinity-norms of switching resolution along trajectory, compared with 2-norm and infinity-norm resolutions.

infinity-norm resolution in the "safe region," and reduces the infinity-norm approximately 23 percent in comparison to 2-norm resolution in the "conservation" region.

VI. Conclusions

The 2-norm and infinity-norm are two popular resolution criteria of redundant systems, and are preferred for different reasons. In order to capture the benefit of both resolution criteria, the previously hypothesized 2-norm/ infinity-norm switching system [4] is implemented in resolving biarticular actuation with equal motor switching levels. The proposed system has three levels of resolution:

1) While all 2-norm resolved motor torques lie within their defined switching levels, the system is resolved using 2-norm. This is analogous to minimizing system energy, when the individual motors are operating in their safe regions.
2) When 2-norm resolves the system with a motor torque in excess of its switching level, the saturated input is truncated and excess contribution is retasked to other, underutilized inputs (CGI resolution).
3) When CGI fails to resolve the system with all inputs within their switching bounds, infinity-norm optimization is utilized. Preventing unnecessary exertion of any particular motor.

This switching system can be implemented without any changes to the existing sensor or actuation structure. Continuity of the proposed switching system was demonstrated, and simulation was conducted demonstrating implementation.

This switching system was proposed and demonstrated with all switching levels equal. In many applications, this will be sufficient as many system designers will choose to use only one motor type when designing their systems. However, the structure of proof should hint at the development of a switching system for more general redundant systems. Future works include evaluation of the potential of such a general switching system, experimental implementation of the proposed switching system including chatter mitigation through switching level hysteresis.

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